

Hard Shadows Aliasing and Remedies

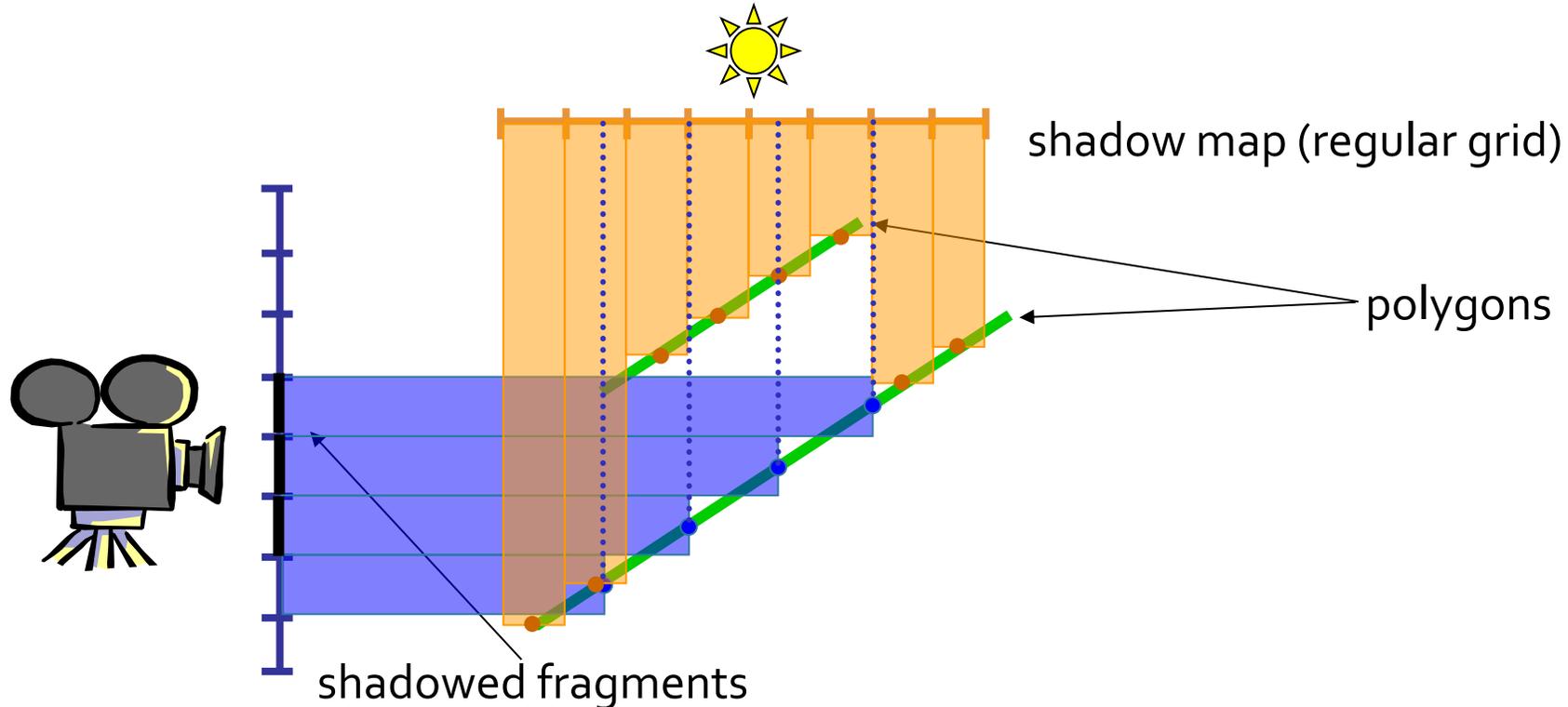
Michael Wimmer

www.realtimeshadows.com



Shadow Map as Signal Reconstruction

- Initial sampling: shadow-map rendering
- Resampling: determined by view
- Reconstruction: nearest neighbor, PCF, ...



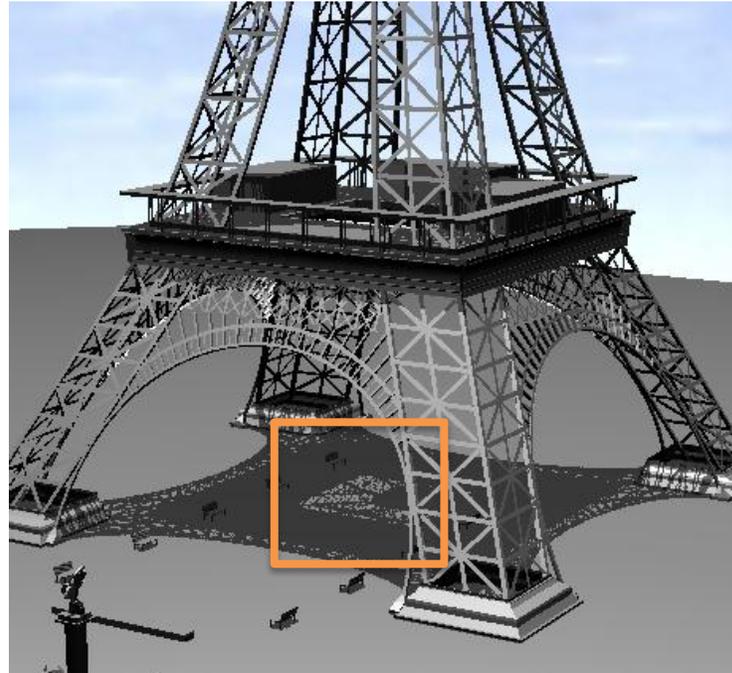
Main Types of Error

Initial sampling:
Undersampling



25 sample Poisson PCF

Resampling:
Oversampling



Reconstruction:
Reconstruction error
(with undersampling)



nearest neighbor

Main Types of Error

■ Undersampling

- No bandlimiting (e.g., „super-sampled shadow map“, possible!)

- Improve initial sampling!

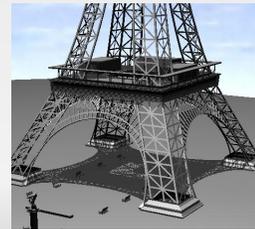


Main topic for next 30 minutes!



■ Oversampling

- Use bandlimiting filters in reconstruction (VSM, CSM, ... → later)

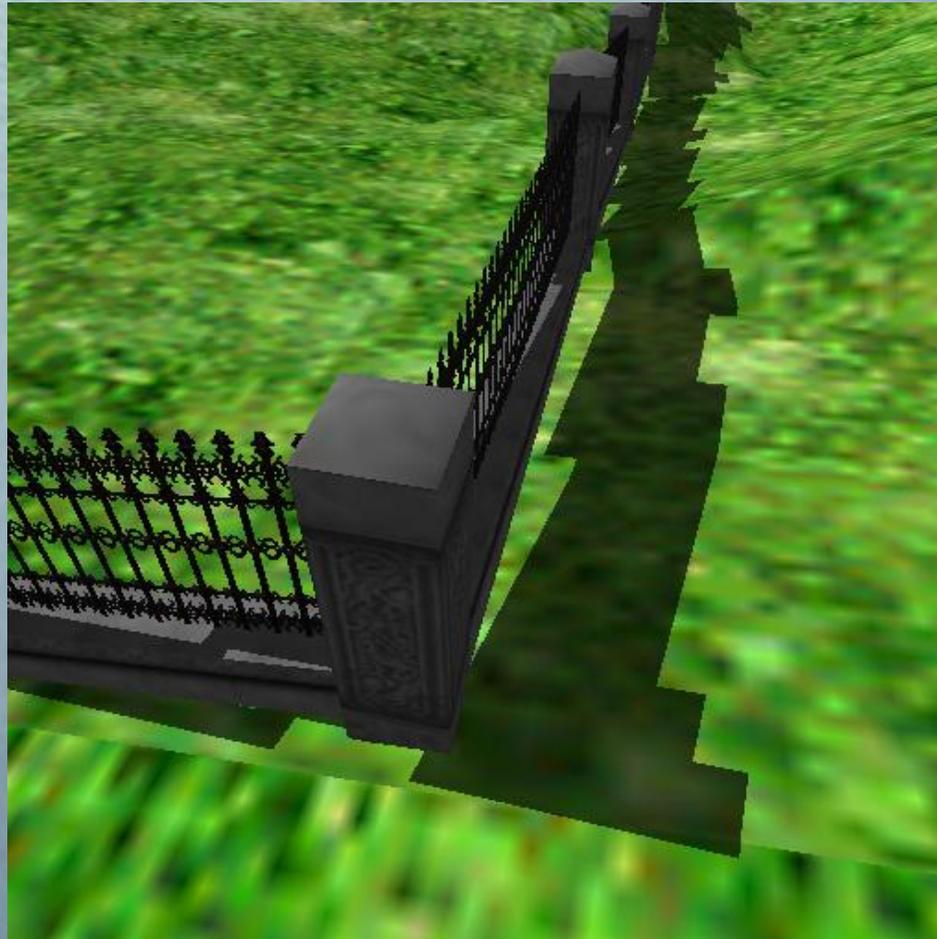


■ Reconstruction error

- Use better reconstruction filters (PCF, ...)
- Use different reconstruction algorithm (Silhouette Shadow Maps)



- Improving Initial Sampling/Undersampling
 - Fitting (Focusing)
 - Error Analysis
 - Warping
 - Partitioning
 - Irregular Sampling
 - Temporal Coherence
- Better reconstruction



Hard Shadows

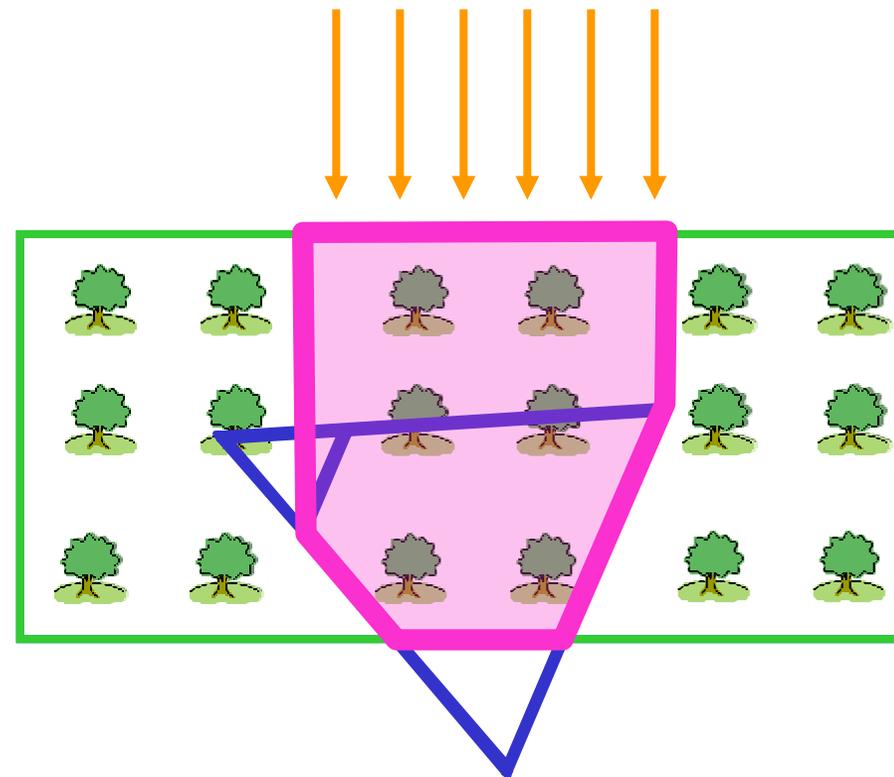
Fighting Undersampling - Fitting

Fitting: Focus the Shadow Map

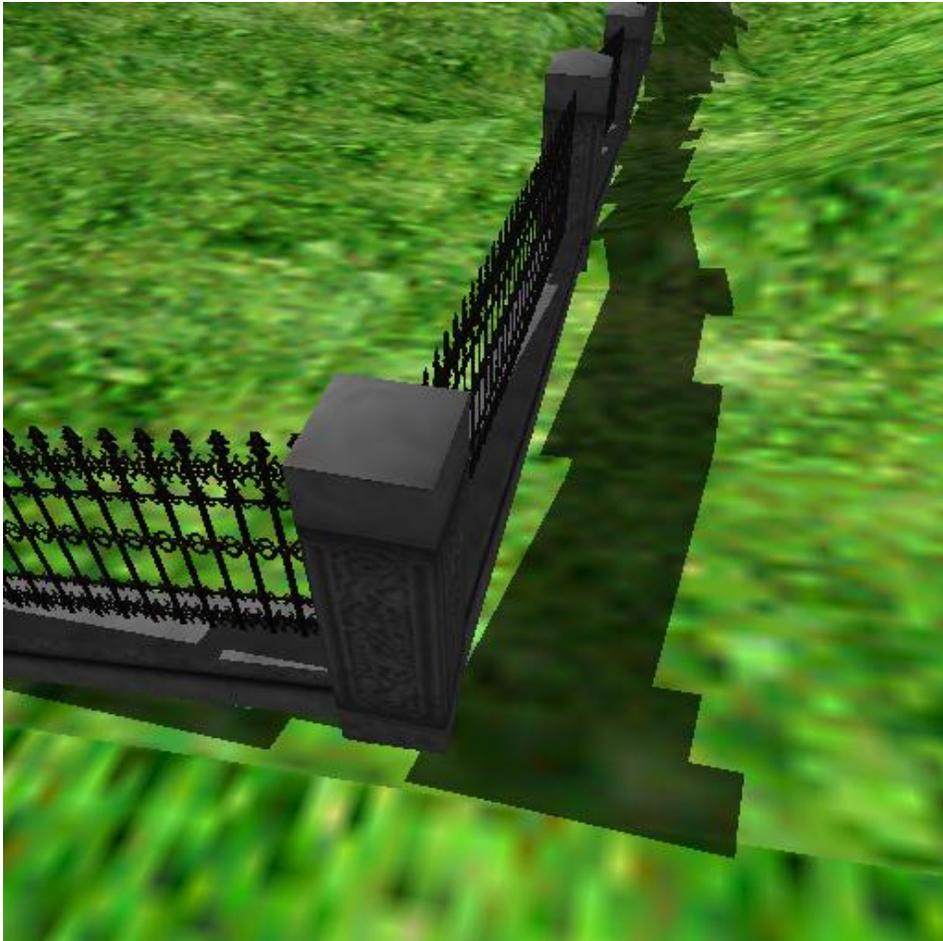
[Brabec et al. 2002]



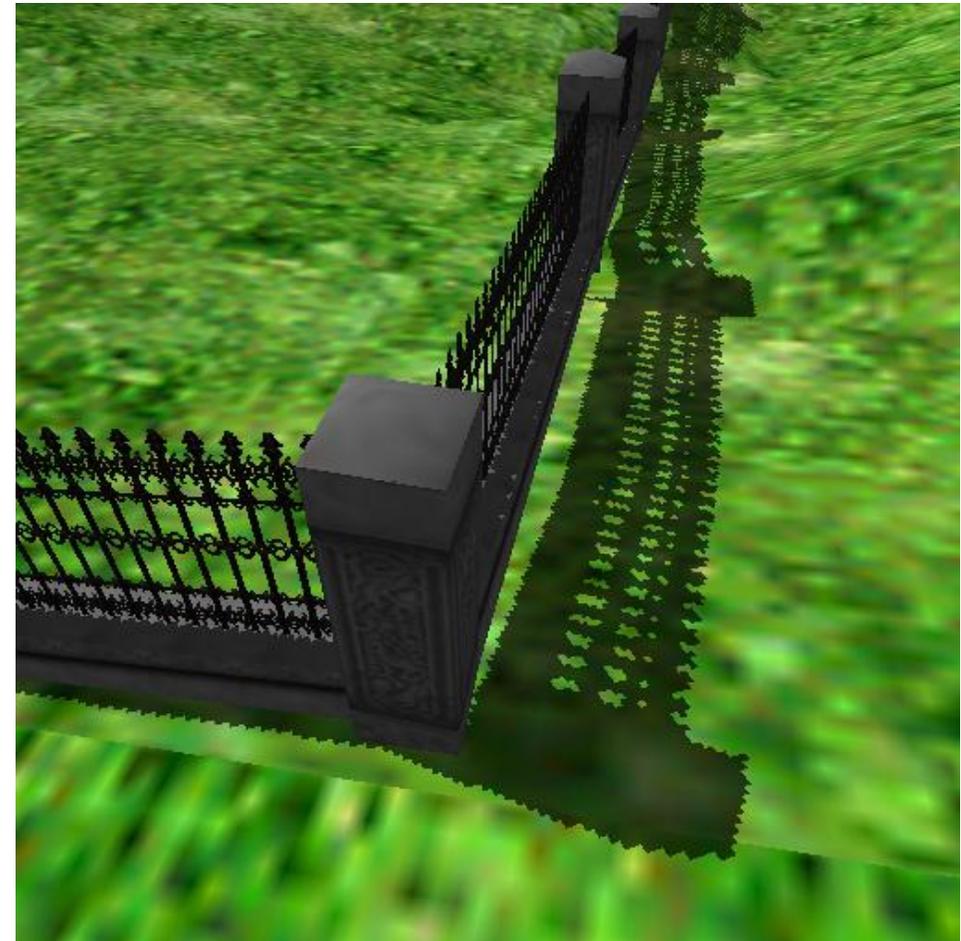
- Only include relevant objects
- Intersection body determined by:
 - Shadow casters
 - Light source frustum
 - View frustum



Fitting: Focus the Shadow Map



Unfocused



Focused

Fitting Calculation

- Model matrix **M**, light view matrix **Lv**, light proj. matrix **Lp**
- Transform intersection body by **Lp Lv**
- Calculate bounds (xmin,ymin,xmax,ymax)
- Calculate fitting matrix **F** („viewport transform“):

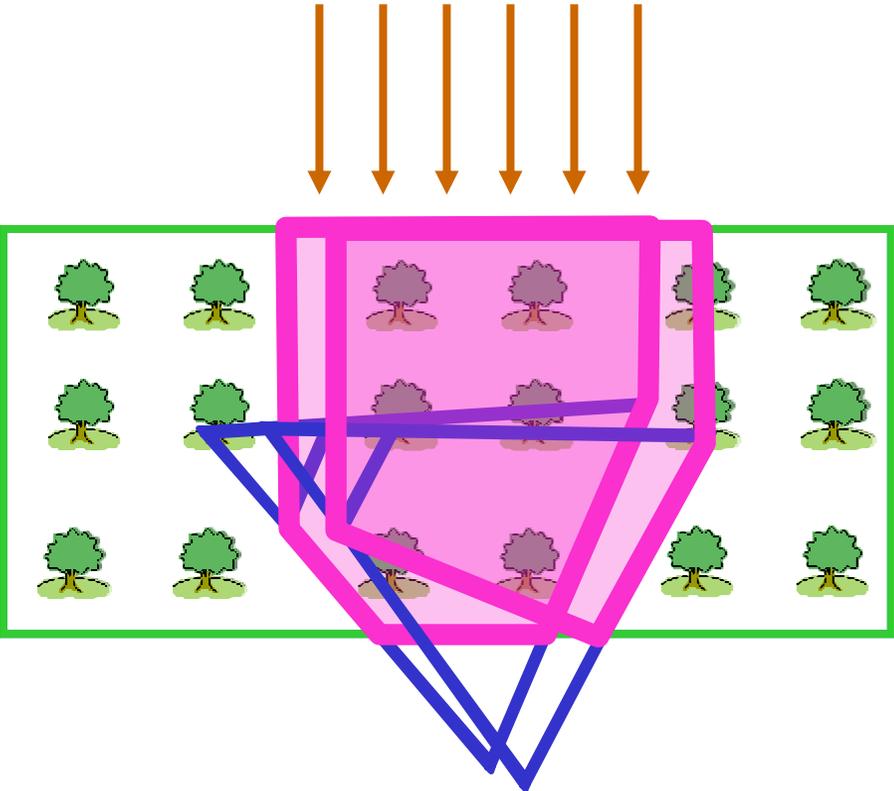
- Shadow matrix:

$$\mathbf{S} = \mathbf{F} \mathbf{Lp} \mathbf{Lv} \mathbf{M}$$

$$\mathbf{F} = \begin{pmatrix} s_x & 0 & 0 & o_x \\ 0 & s_y & 0 & o_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$s_x = \frac{2}{x_{\max} - x_{\min}}, \quad o_x = -\frac{s_x (x_{\max} + x_{\min})}{2}$$

Fitting: Temporal Aliasing



Fitting: Temporal Aliasing

■ Solutions:

- Increase initial sampling frequency
- Make fitting adhere to texel boundaries:

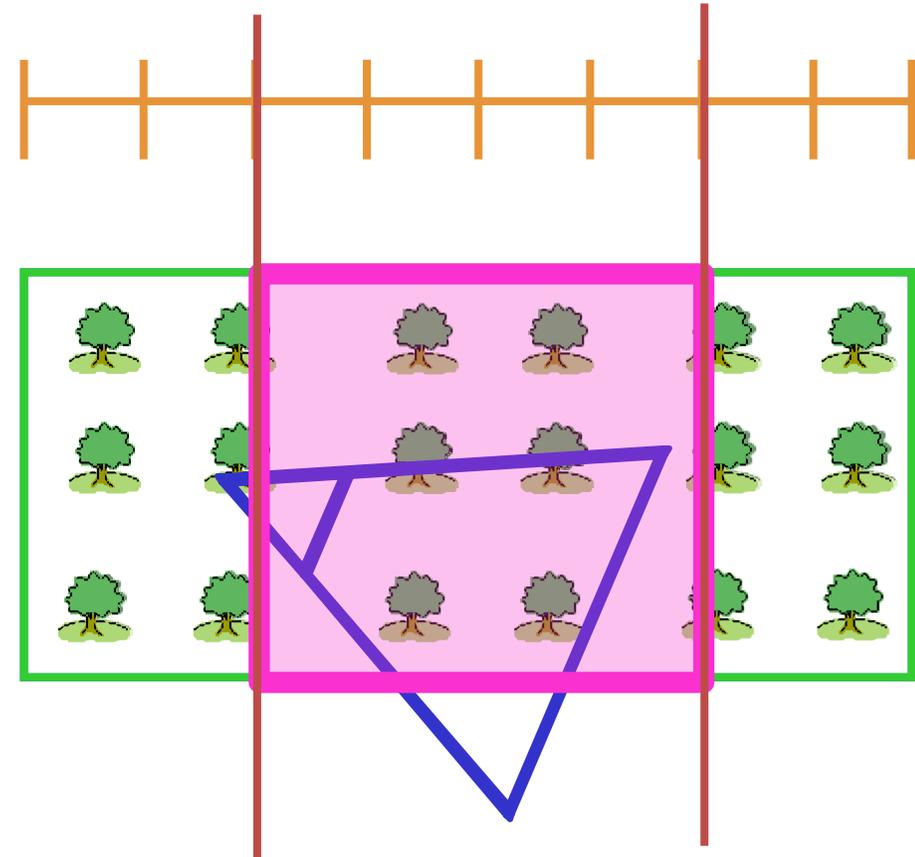
- Adjust fitting matrix **F**:

- r : half shadow-map resolution

- o : ox/oy

$$o' = \frac{\text{ceil}(or)}{r}$$

- Will not work for warping



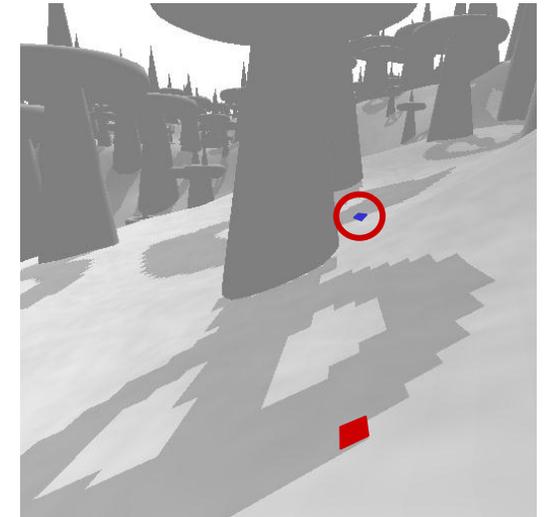
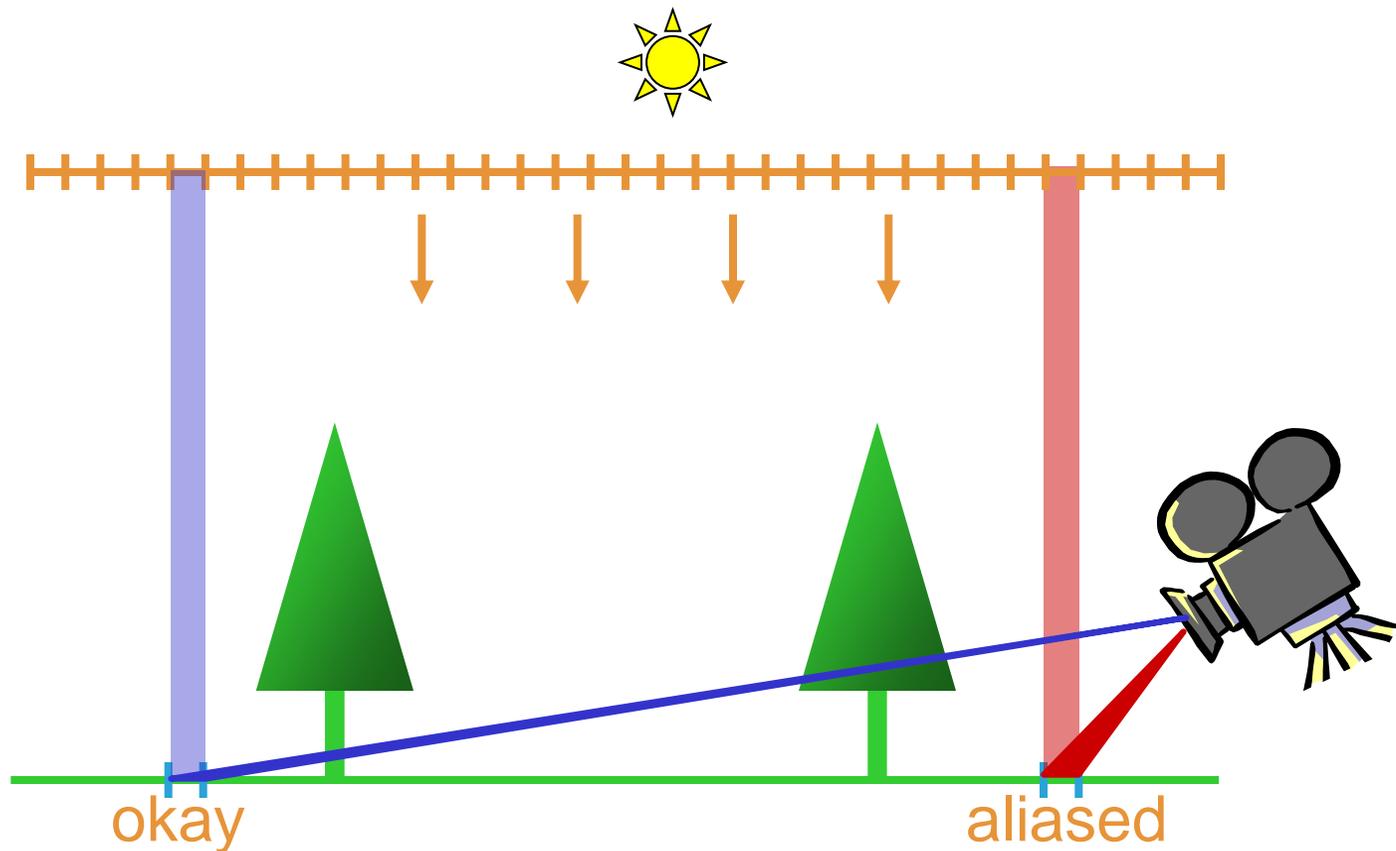
$$\frac{dp}{ds} = \frac{z_n}{z} \frac{dz}{ds} \frac{\cos \alpha}{\cos \beta}$$

Hard Shadows

Error Analysis

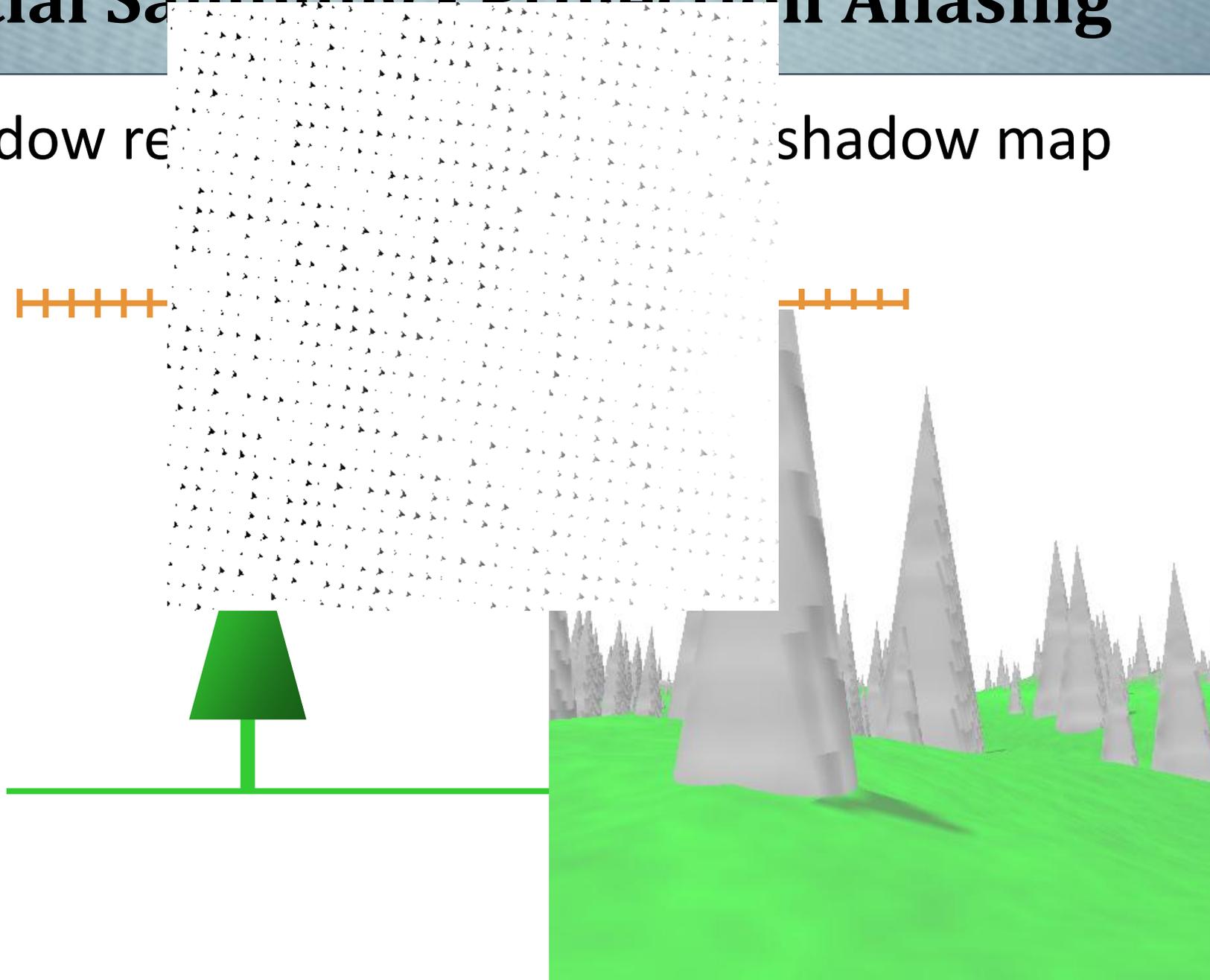
Initial Sampling - Perspective Aliasing

- **Sufficient** resolution far from the observer
- **Insufficient** resolution near the observer



Initial Sampling - Projection Aliasing

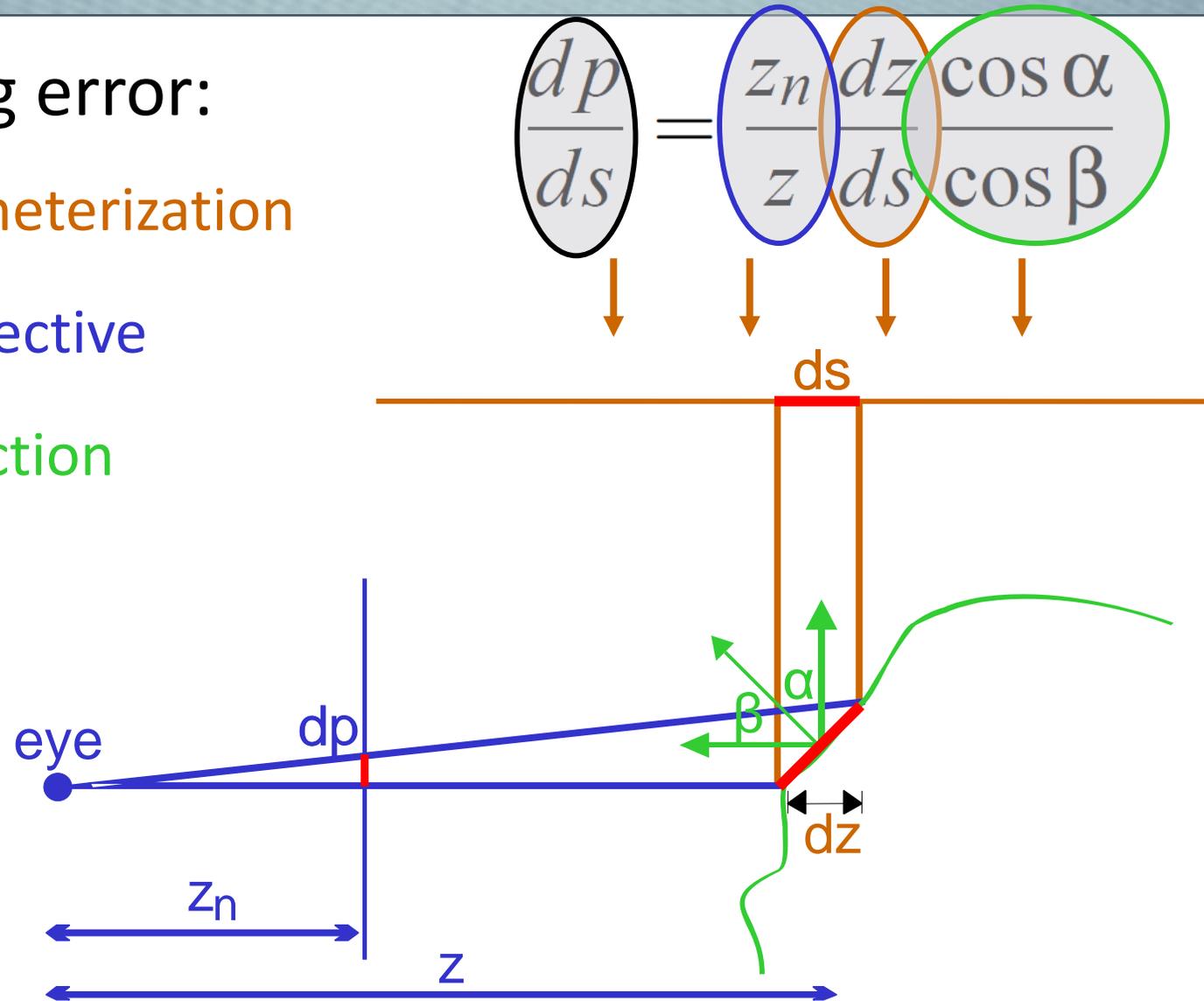
- Shadow rendering of a scene with a shadow map

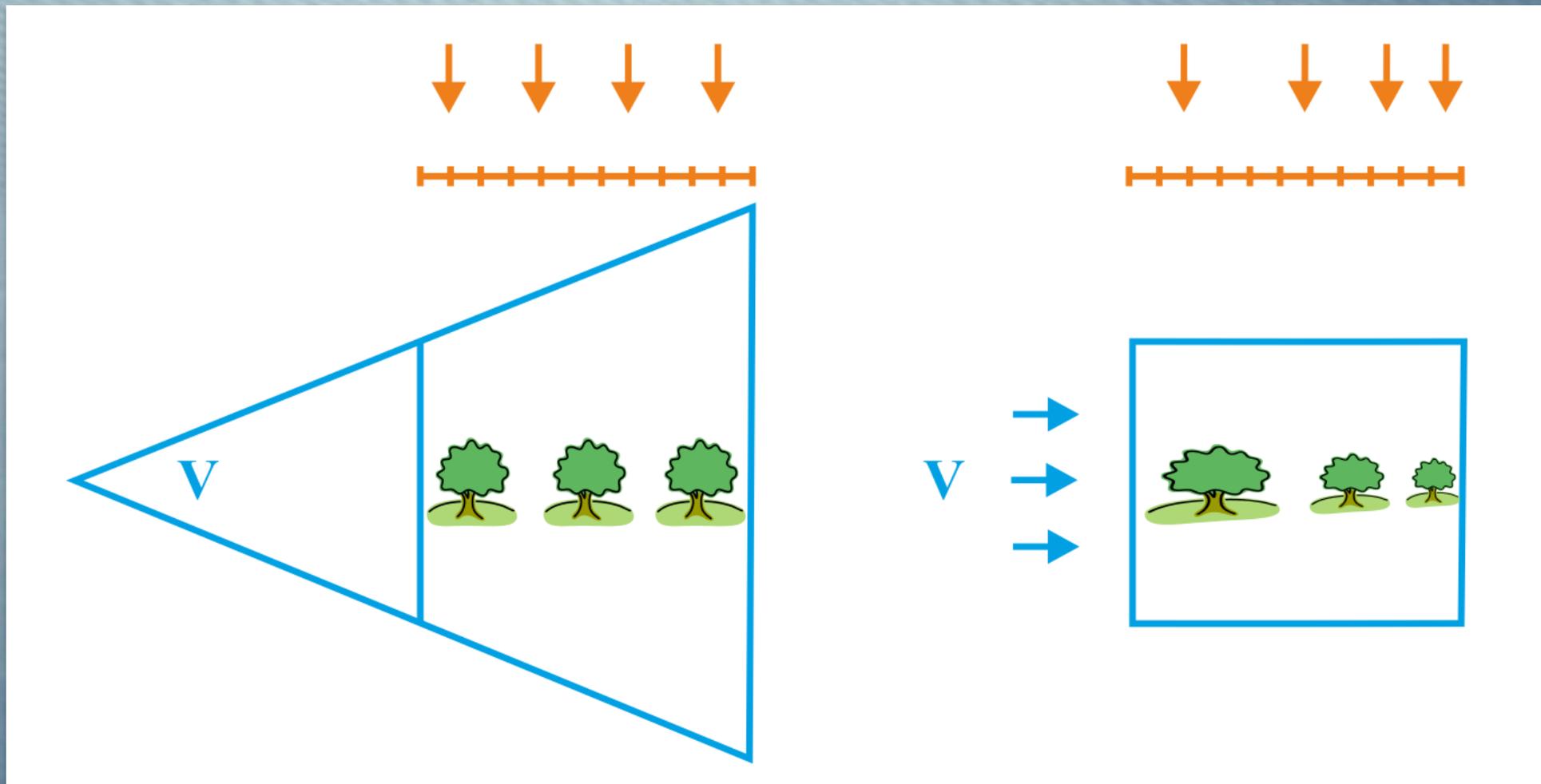


(Simple) Initial Sampling Error Analysis

■ Aliasing error:

- Parameterization
- Perspective
- Projection



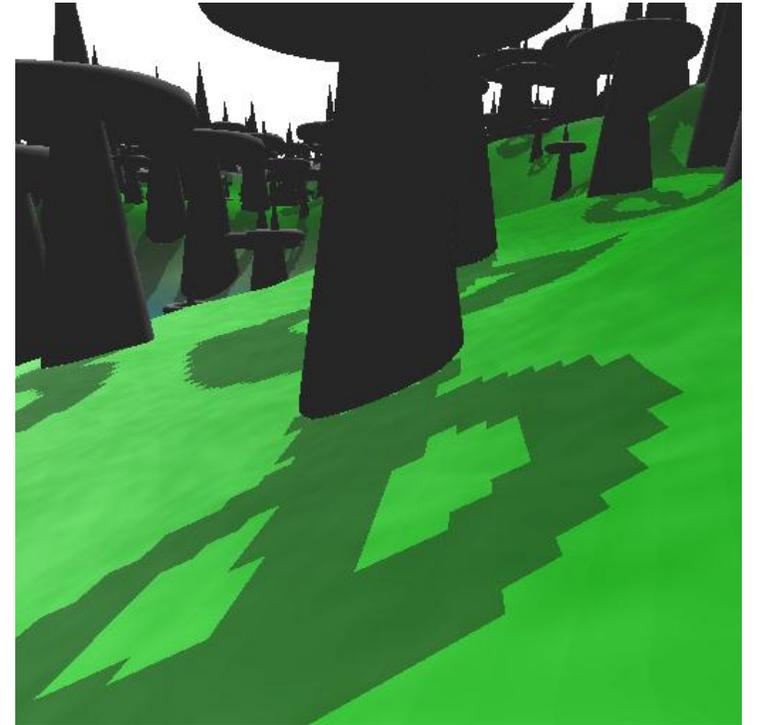
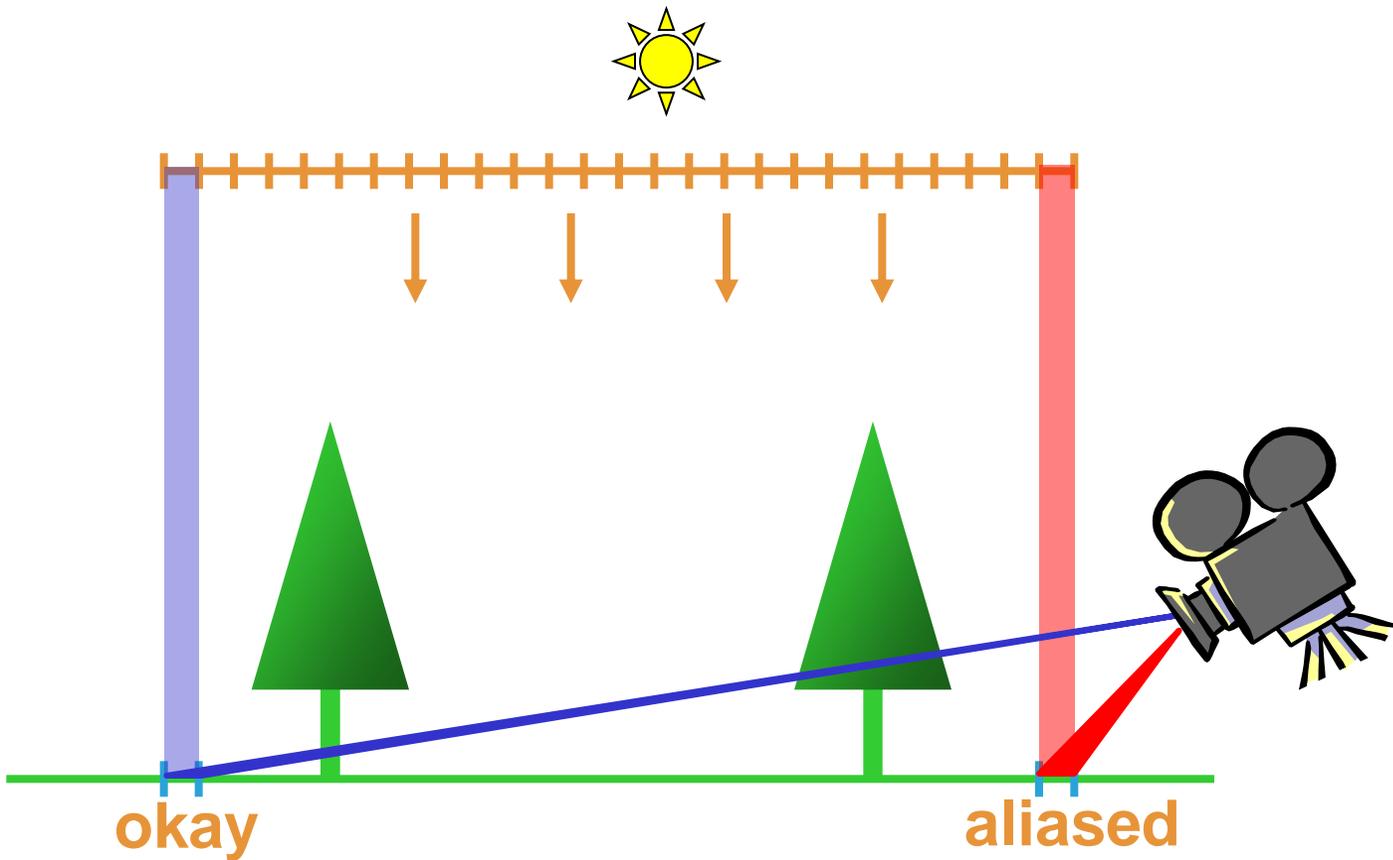


Hard Shadows

Fighting Undersampling – Warping

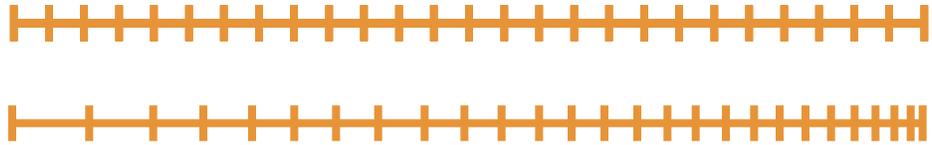
Solution for Perspective Aliasing

- **Insufficient** resolution near eye



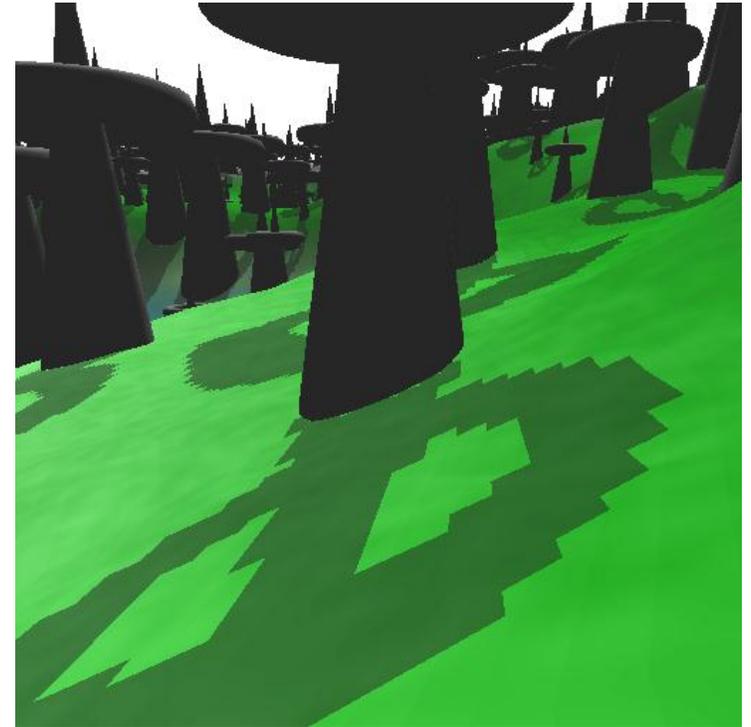
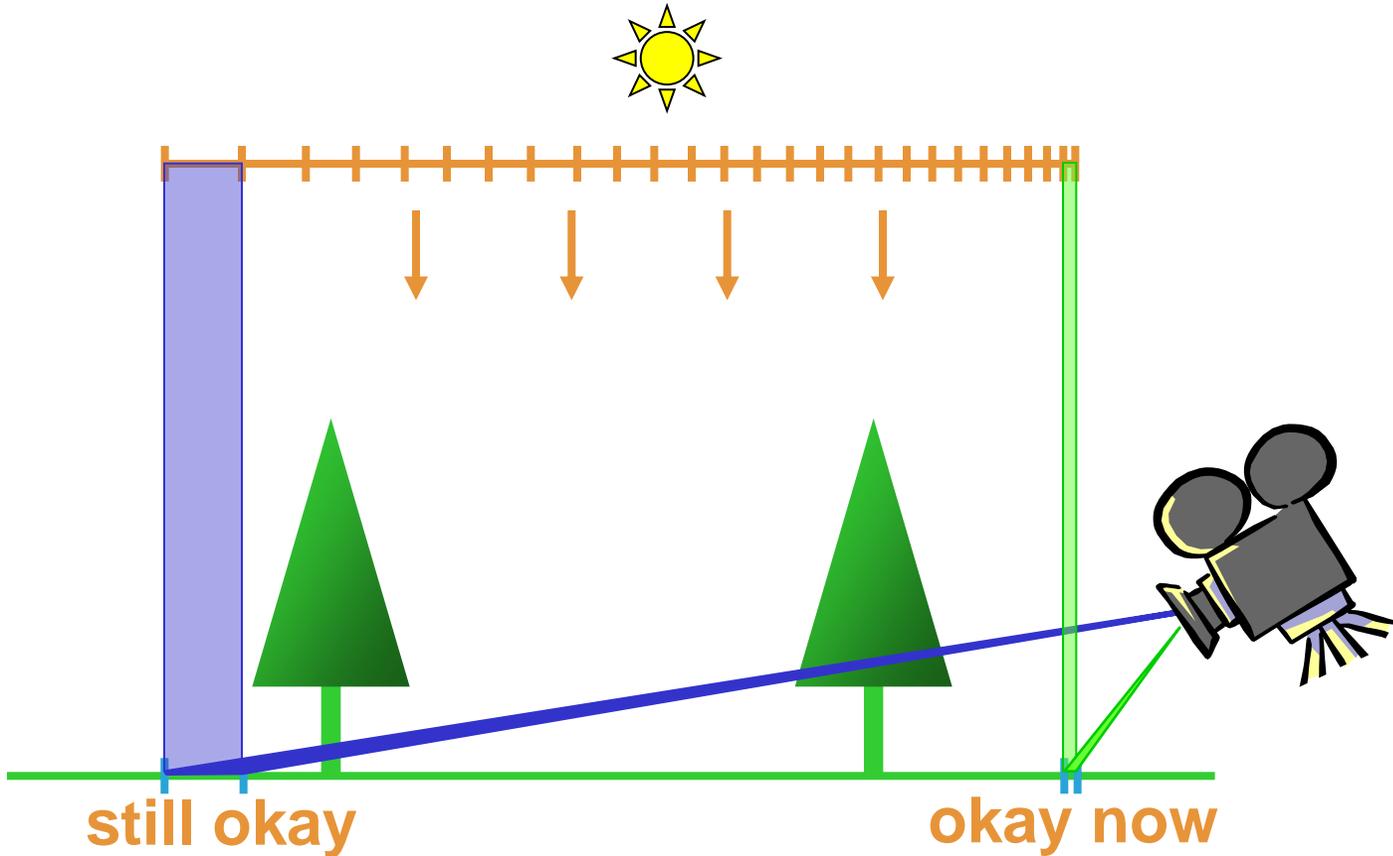
Solution for Perspective Aliasing

- **Insufficient** resolution near eye
- **Redistribute** values in shadow map



Solution for Perspective Aliasing

- **Sufficient** resolution near eye
- **Redistribute** values in shadow map

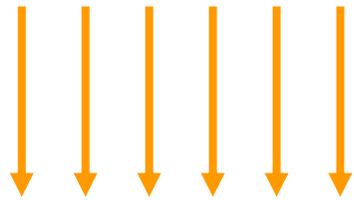


Shadow Map Warping

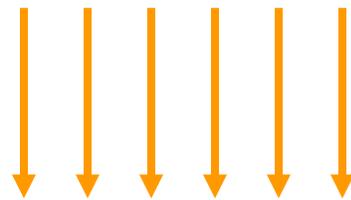
- Use an **additional perspective frustum**

- **Perspective Shadow Maps (PSM)** [Stamminger & Drettakis 2002]

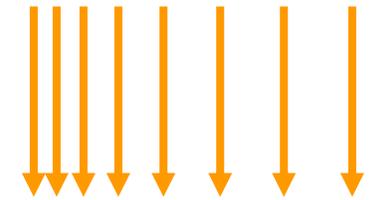
uniform shadow map



warped scene



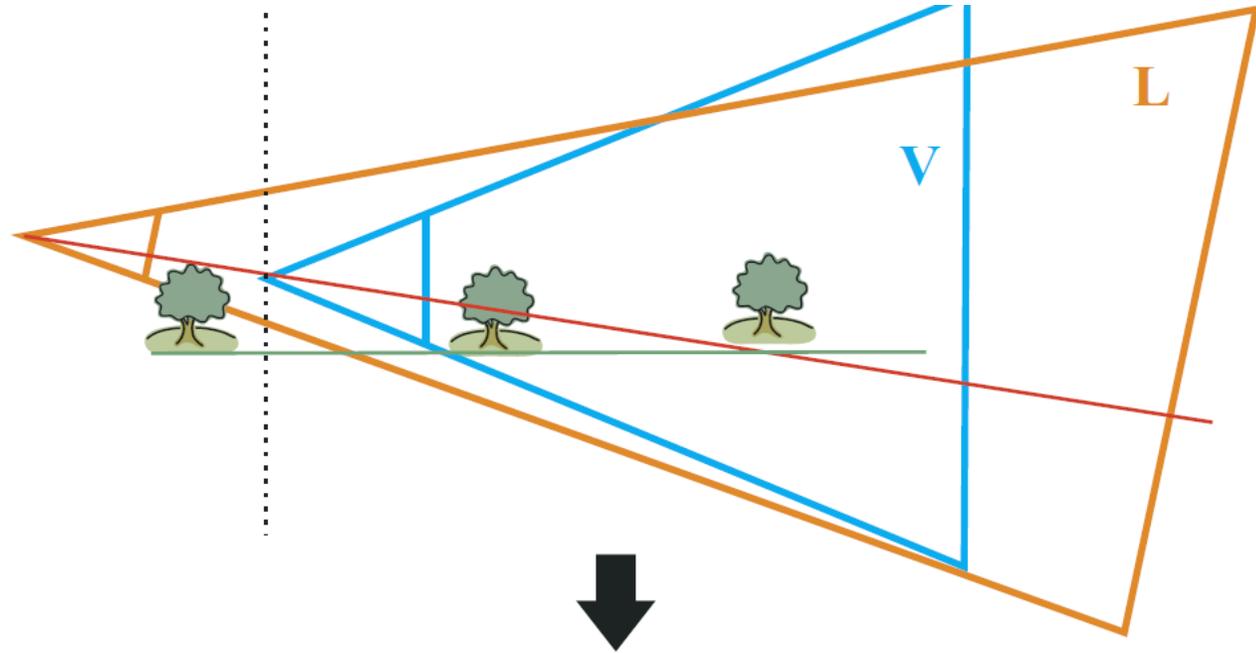
warped shadow map



PSM: Warping Frustum = View Frustum!

→ Shadows from behind

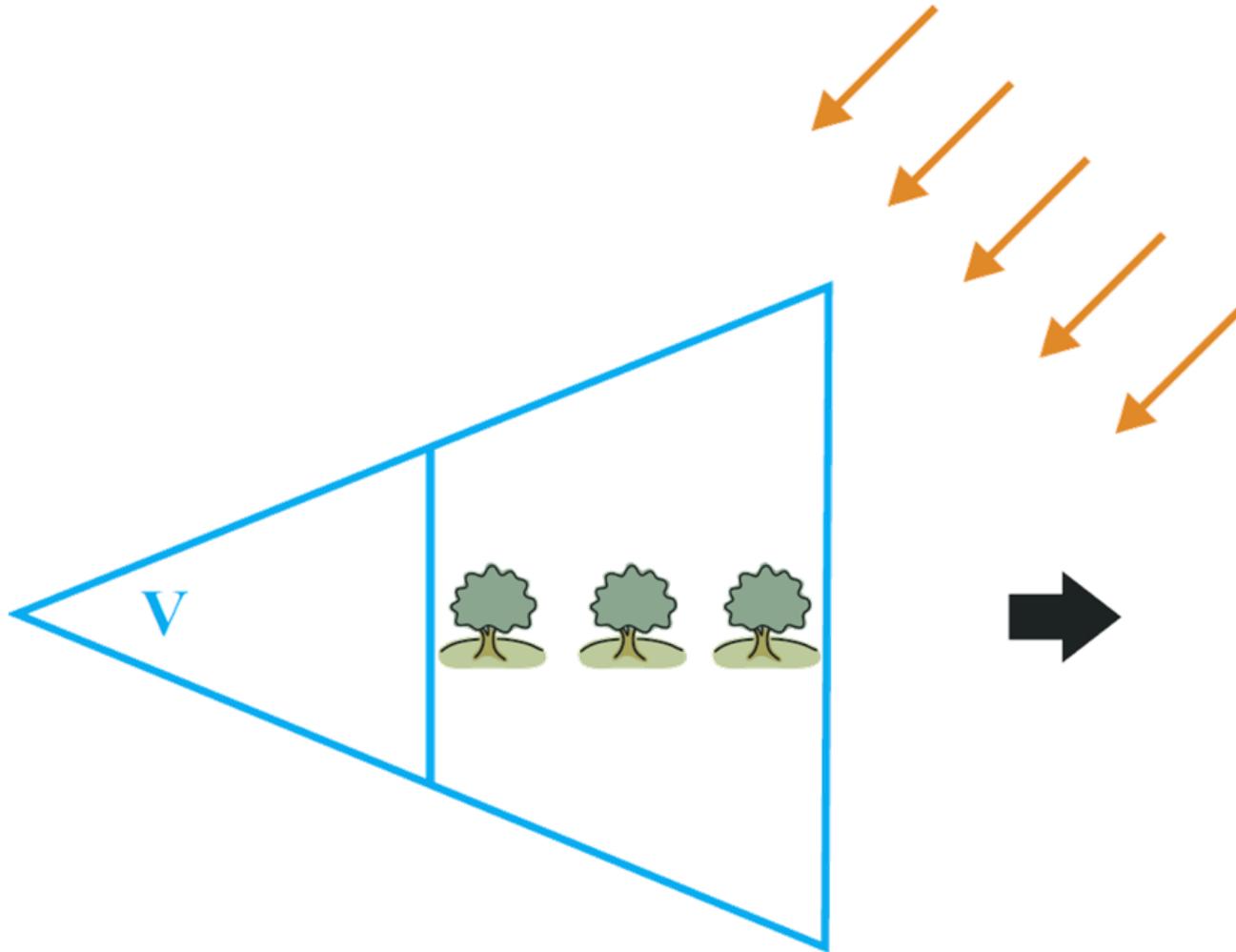
world space



post-perspective space

PSM: Warping Frustum = View Frustum!

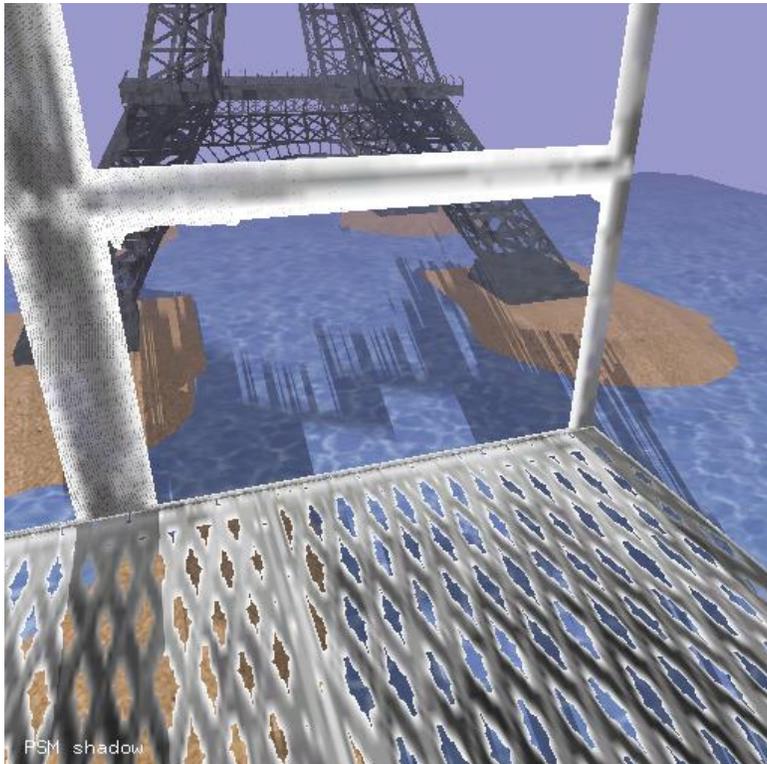
→ Light changes type



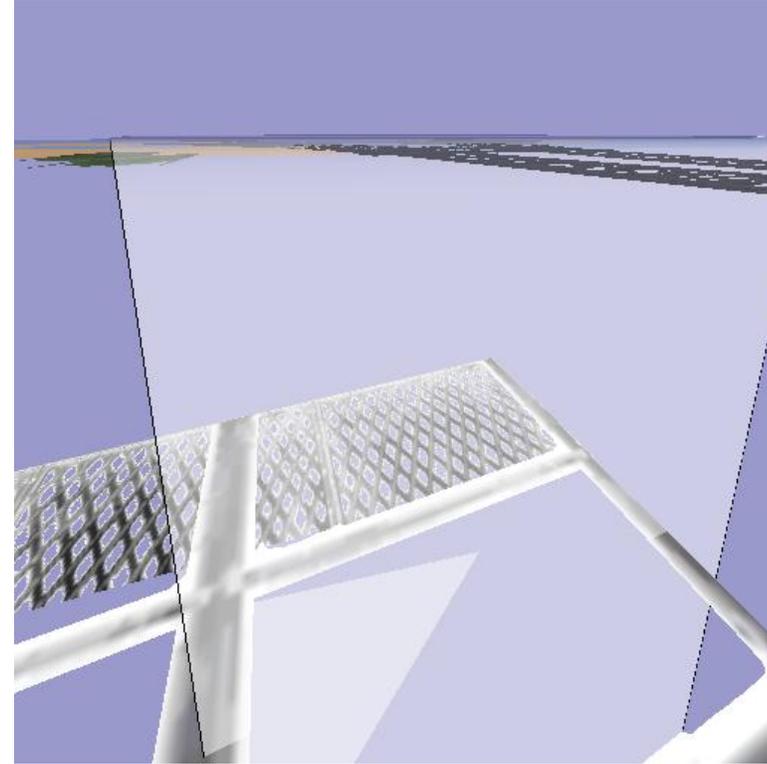
PSM: Warping Frustum = View Frustum!

→ Most severe: uneven z-distribution

- Good near viewer, very bad far away!
- Can be reduced by pushing near plane away



world space



post-perspective space

■ Enter **Light-Space Perspective Shadow Maps (LiSPSM)**

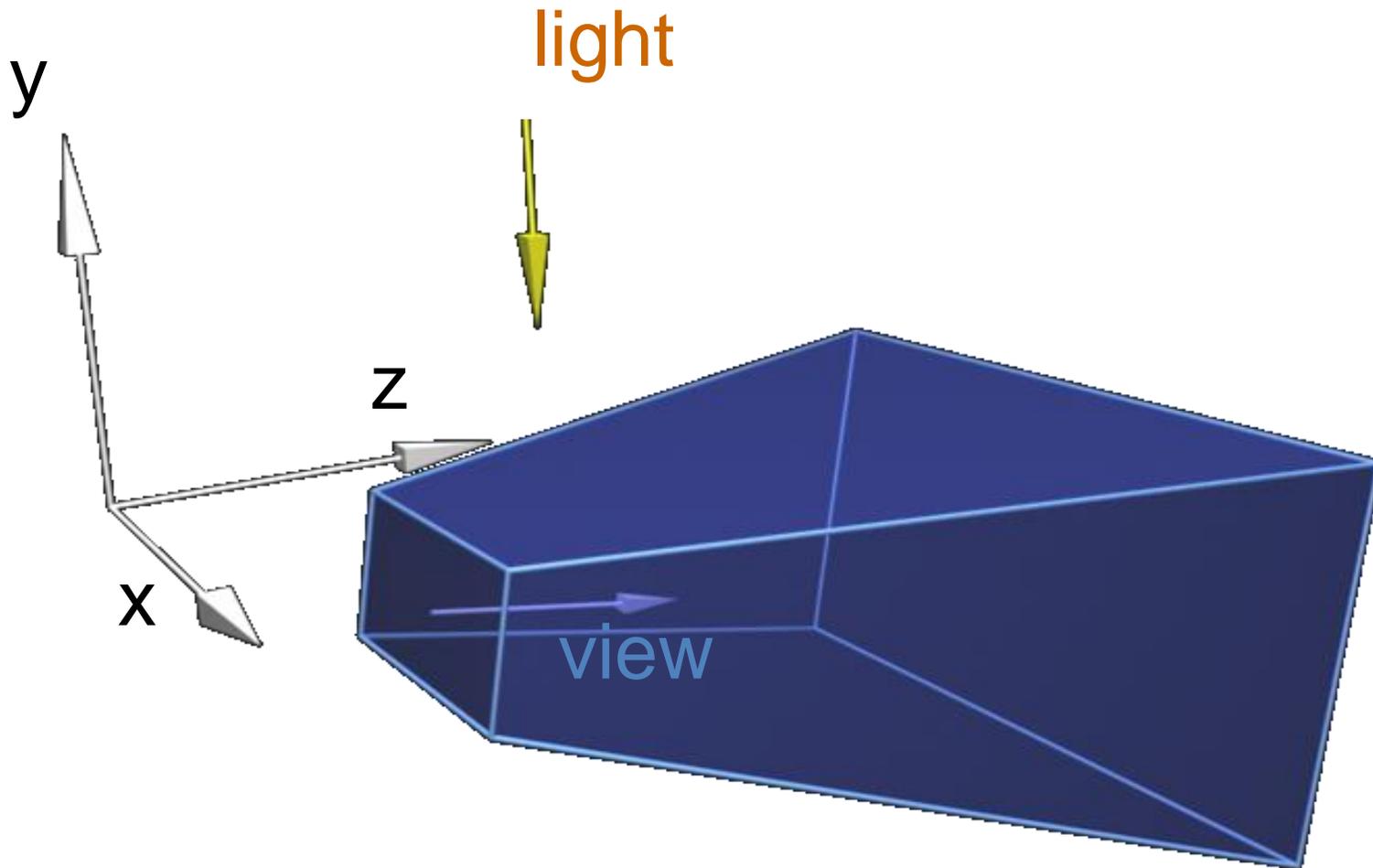
[Wimmer et al. 2004]

- Perspective frustum defined relative to light space
- Optimize warping strength to improve z-distribution
- Similar to Trapezoidal Shadow Maps (TSM)

[Martin & Tan 2004]

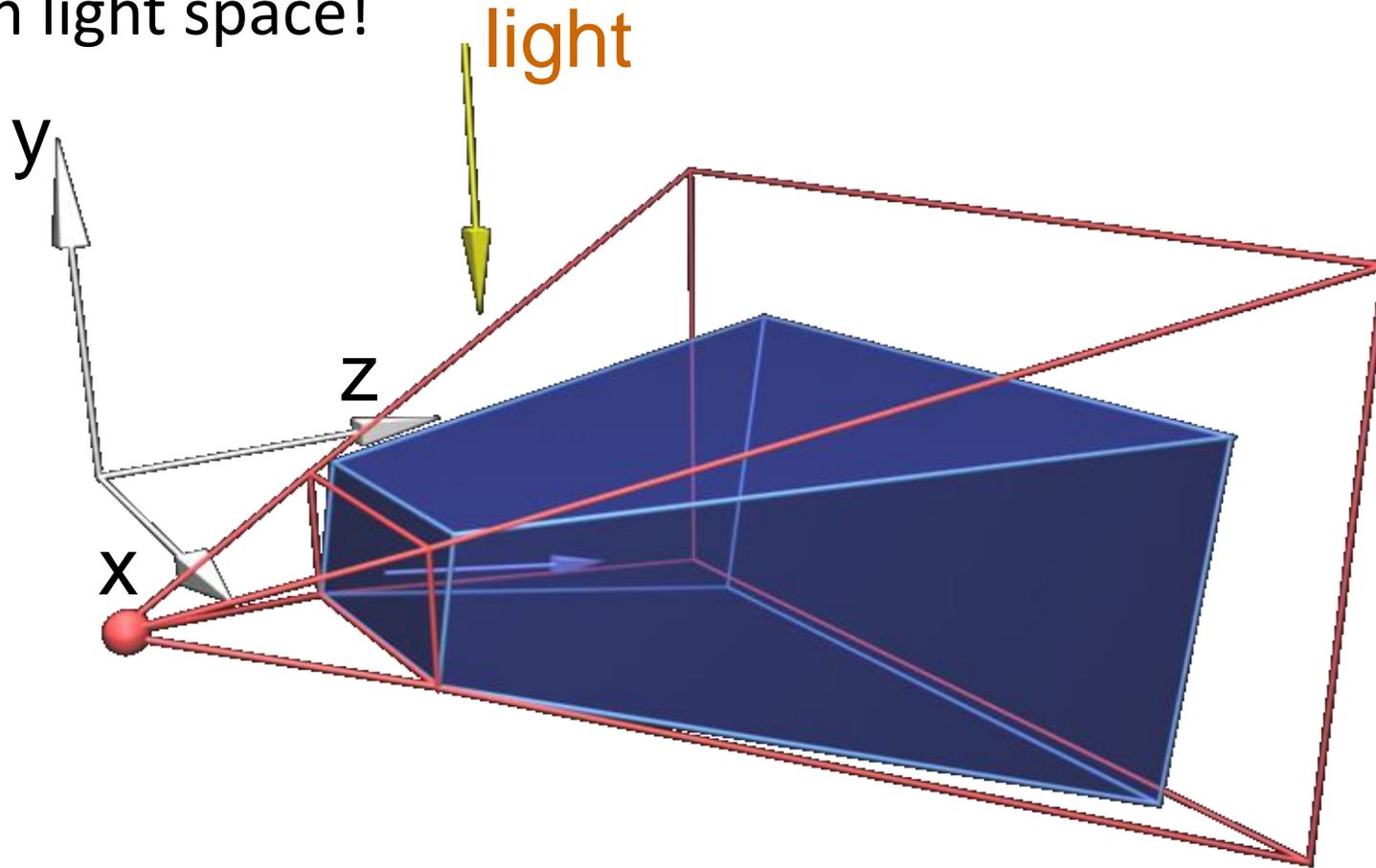
Light-Space Perspective Shadow Mapping (LiSPSM)

- **Light** and **view** vector define yz-plane



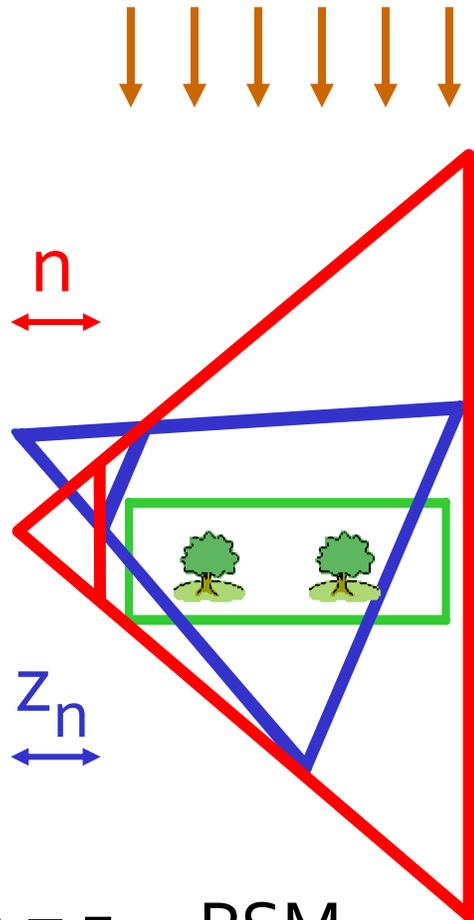
Light-Space Perspective Shadow Mapping (LiSPSM)

- Find a **tight perspective frustum** on focused region
 - In light space!

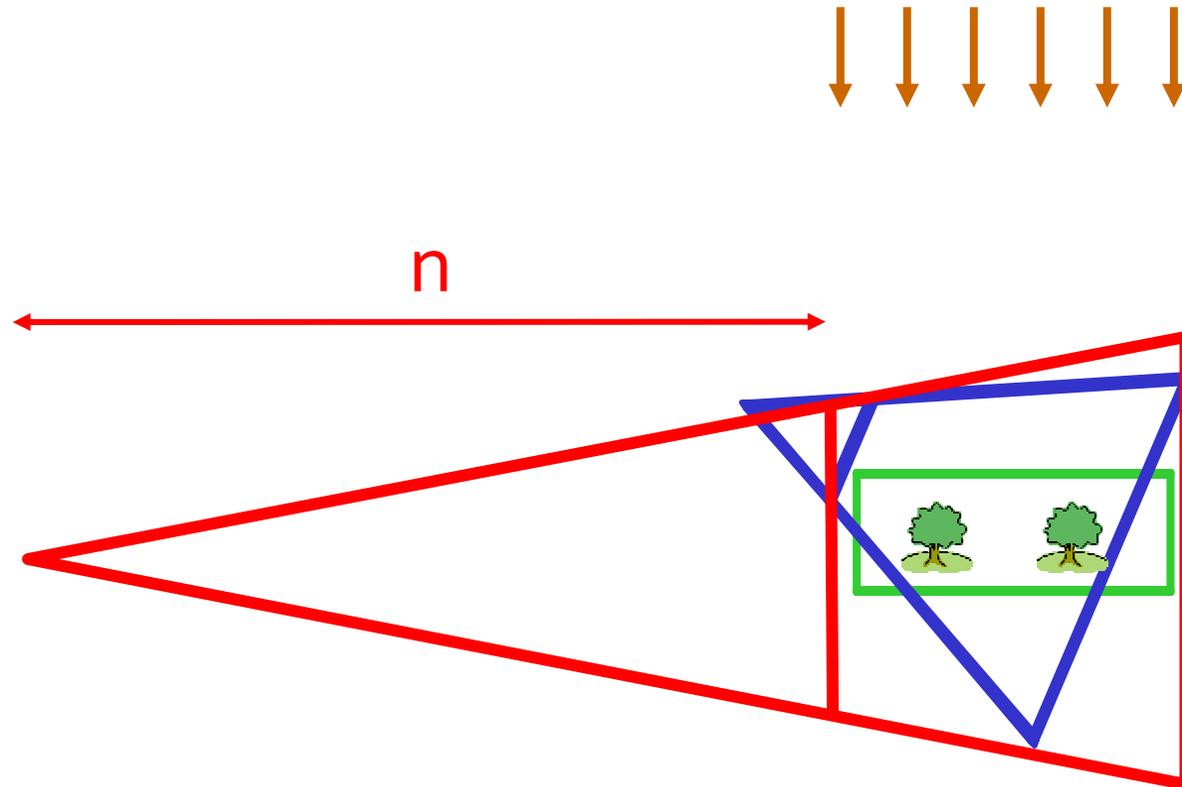


Free Parameter n

- Controls warping effect

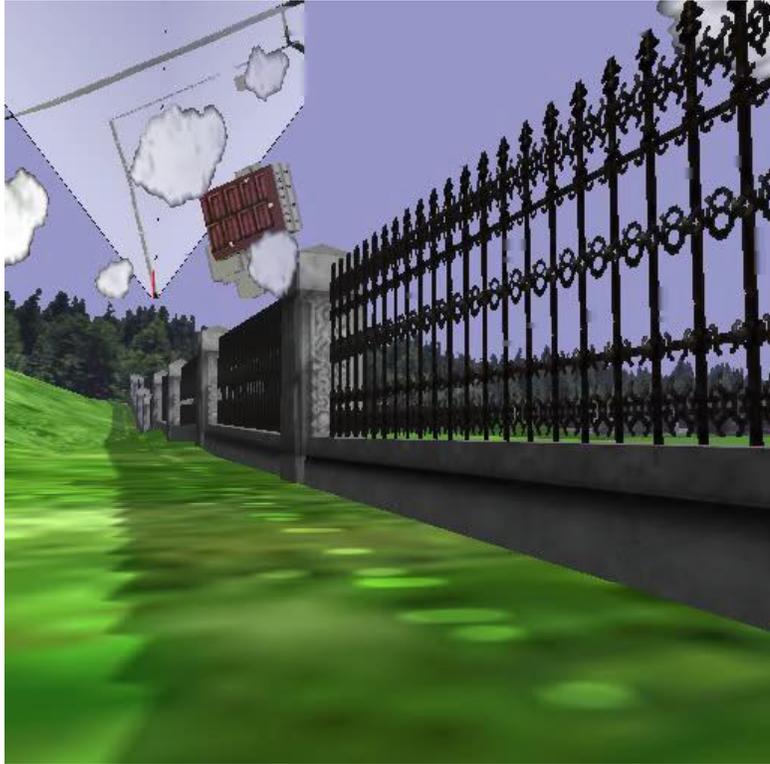


$n = z_n$: PSM

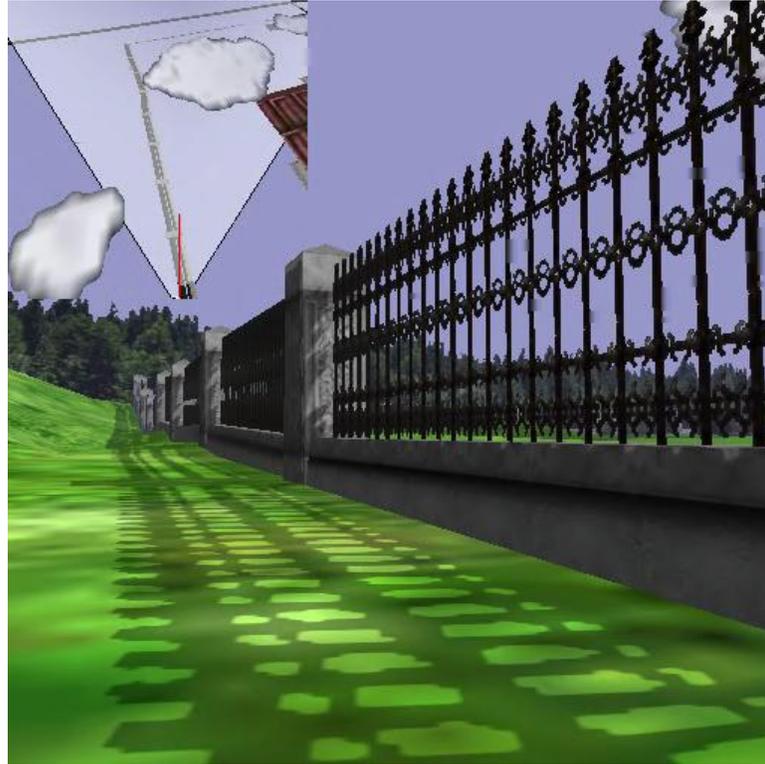


$n \rightarrow \text{infinity}$: uniform shadow maps

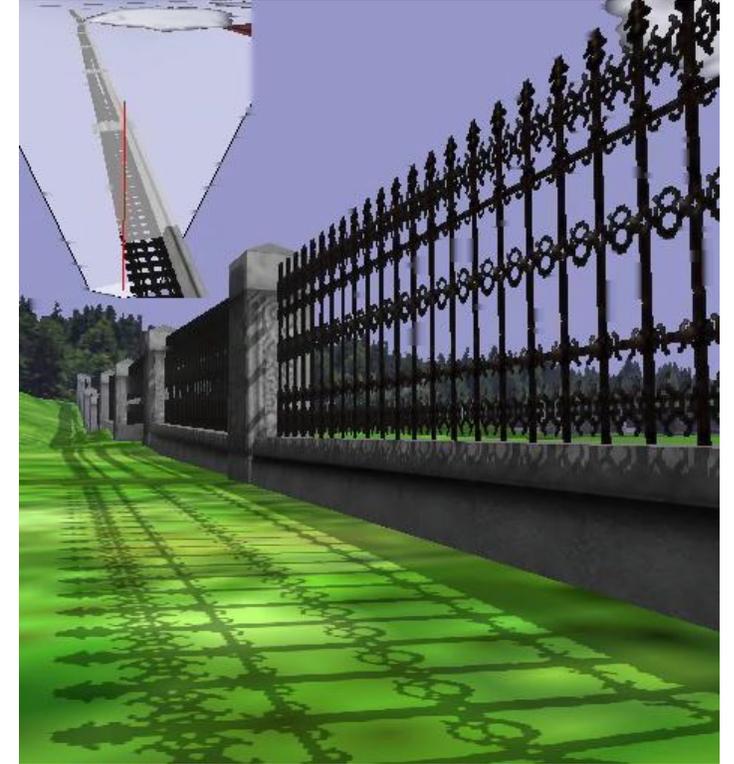
Free Parameter n



very big n



optimal n ?



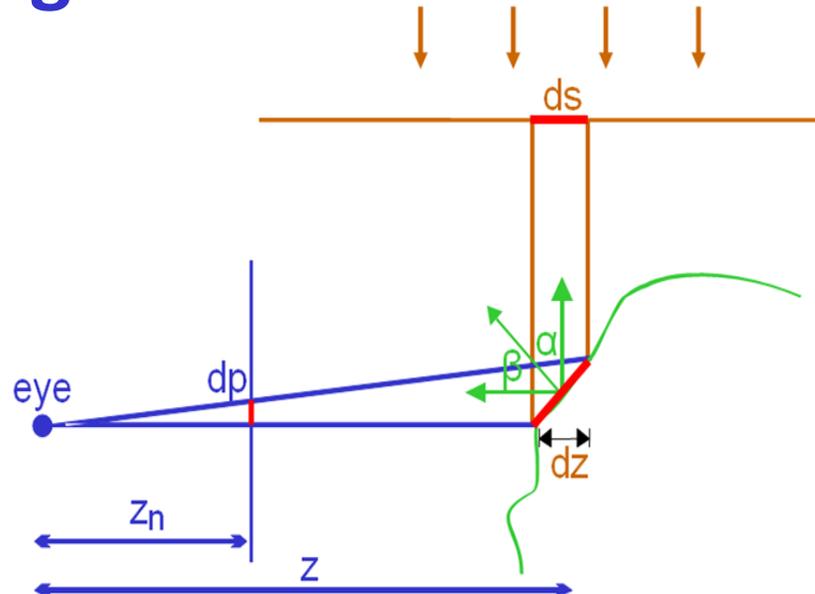
very small n

- Standard light matrix: $\mathbf{S} = \mathbf{F} \mathbf{L}_p \mathbf{L}_v \mathbf{M}$
- Orient shadow map along view vector: \mathbf{R}
- Transform intersection body by $\mathbf{R} \mathbf{L}_p \mathbf{L}_v$
- Find near/far, and choose warping strength n
- Calculate warping matrix: $\mathbf{W}_p \mathbf{W}_v$
- Calculate \mathbf{F} using $\mathbf{W}_p \mathbf{W}_v \mathbf{R} \mathbf{L}_p \mathbf{L}_v$
- Warped shadow matrix $\mathbf{S}_w = \mathbf{F} \mathbf{W}_p \mathbf{W}_v \mathbf{R} \mathbf{L}_p \mathbf{L}_v \mathbf{M}$
- Use \mathbf{S}_w both for shadow-map generation and rendering

How to Choose the Free Parameter n ?

- Recall error analysis
- $\frac{dp}{ds} > 1 \rightarrow$ shadow map undersampling
- **Projection aliasing** cannot be changed
- Counter **perspective aliasing** with new shadow map **parameterization** $s(z, \dots)$
- Goal: $\frac{dp}{ds} \sim 1$

$$\frac{dp}{ds} = \left(\frac{z_n}{z} \right) \left(\frac{dz}{ds} \right) \left(\frac{\cos \alpha}{\cos \beta} \right)$$



Error Analysis

$$\frac{dp}{ds} = \frac{z_n}{z} \frac{dz}{ds} \frac{\cos \alpha}{\cos \beta}$$



- Perfect: logarithmic re-parameterization

$$s \sim \log z \quad \Rightarrow \quad \frac{ds}{dz} \sim \frac{1}{z} \quad \Rightarrow \quad \frac{dp}{ds} \sim 1$$

- Hardware support? [Lloyd 2007, 2008]

- Uniform shadow maps

$$s \sim z \quad \Rightarrow \quad \frac{ds}{dz} \sim 1 \quad \Rightarrow \quad \frac{dp}{ds} \sim \frac{1}{z}$$

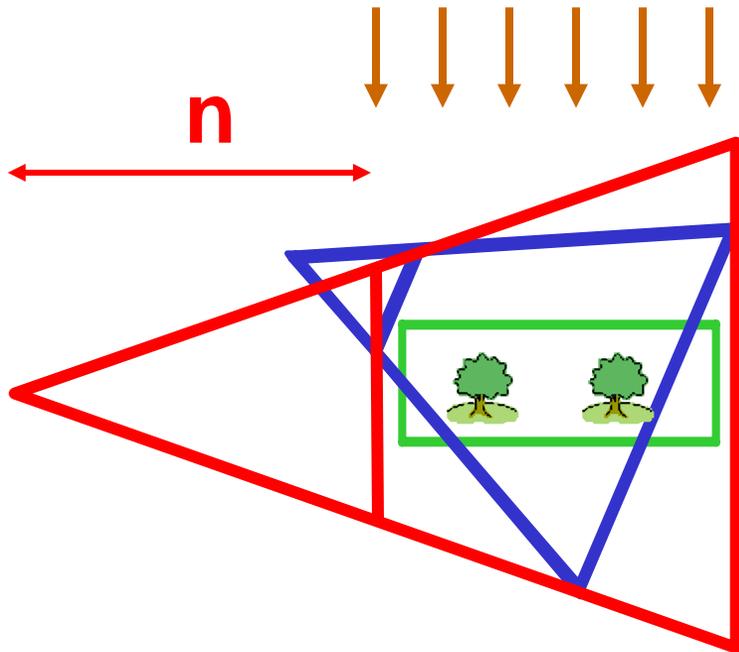
- Perspective shadow maps

$$s \sim \frac{1}{z} \quad \Rightarrow \quad \frac{ds}{dz} \sim \frac{1}{z^2} \quad \Rightarrow \quad \frac{dp}{ds} \sim z$$

- Linear increase in error!

Error Analysis: LiSPSM Optimal Choice

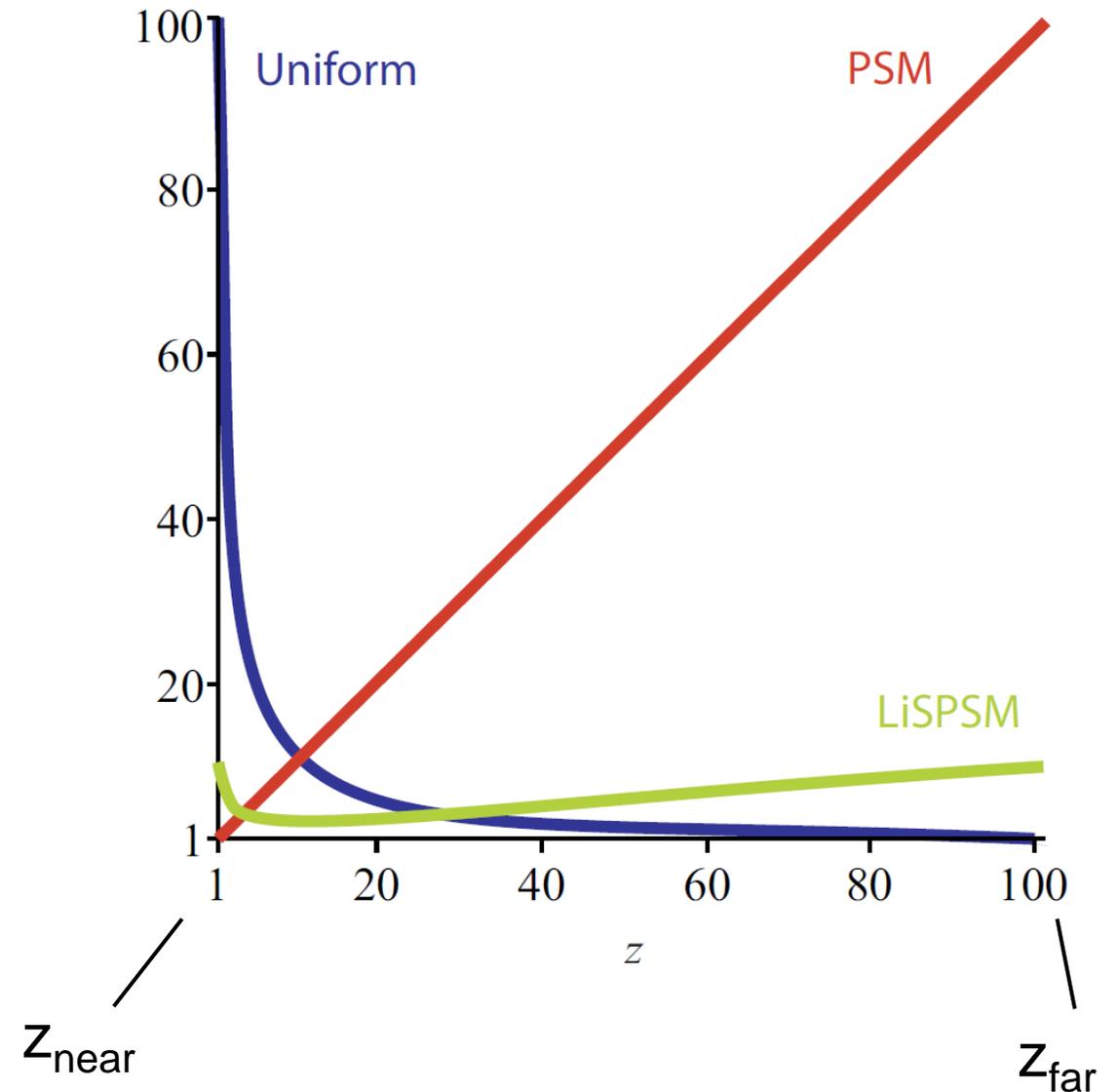
- For LiSPSM, $\frac{dp}{ds}$ depends on n
 - Gives $\frac{dp}{ds}$ between uniform and perspective
- Optimal choice:



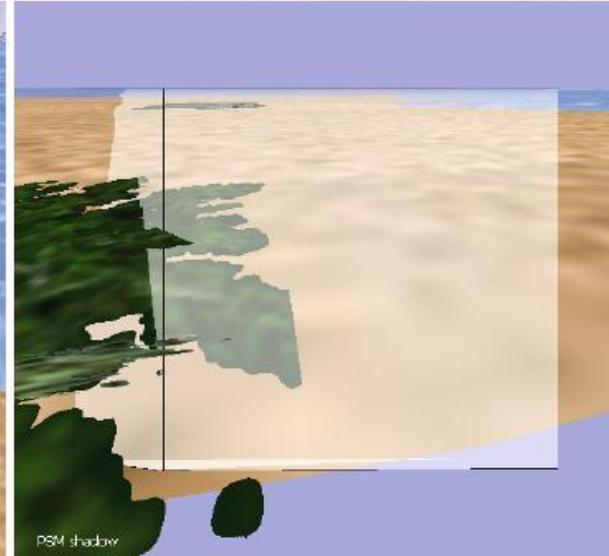
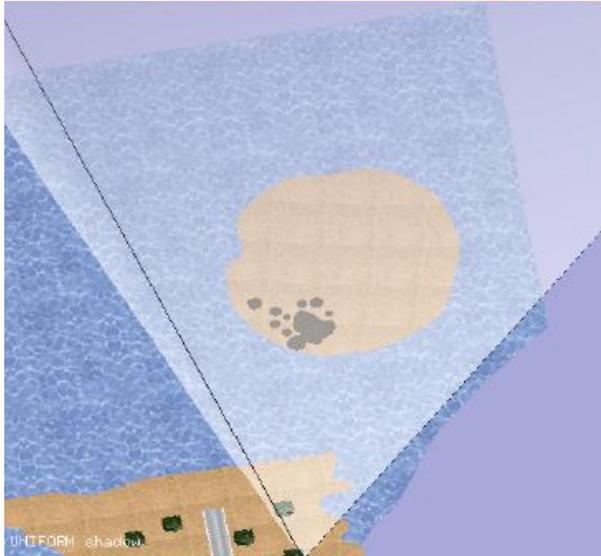
$$n_{opt} = z_n + \sqrt{z_f z_n}$$

Error Comparison

- LiSPSM optimal choice
- Measured along view dir
- LiSPSM vs PSM
→ for same depth range, LiSPSM error much lower



Comparison



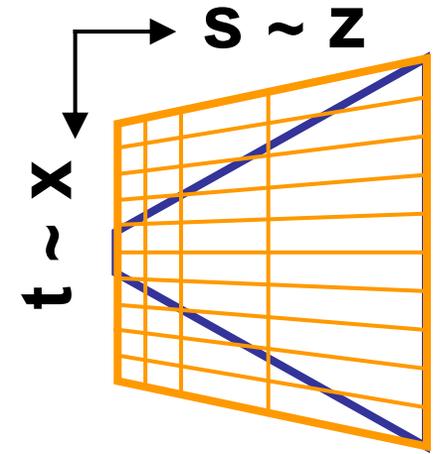
Uniform

LiSPSM

PSM

Error Comparison

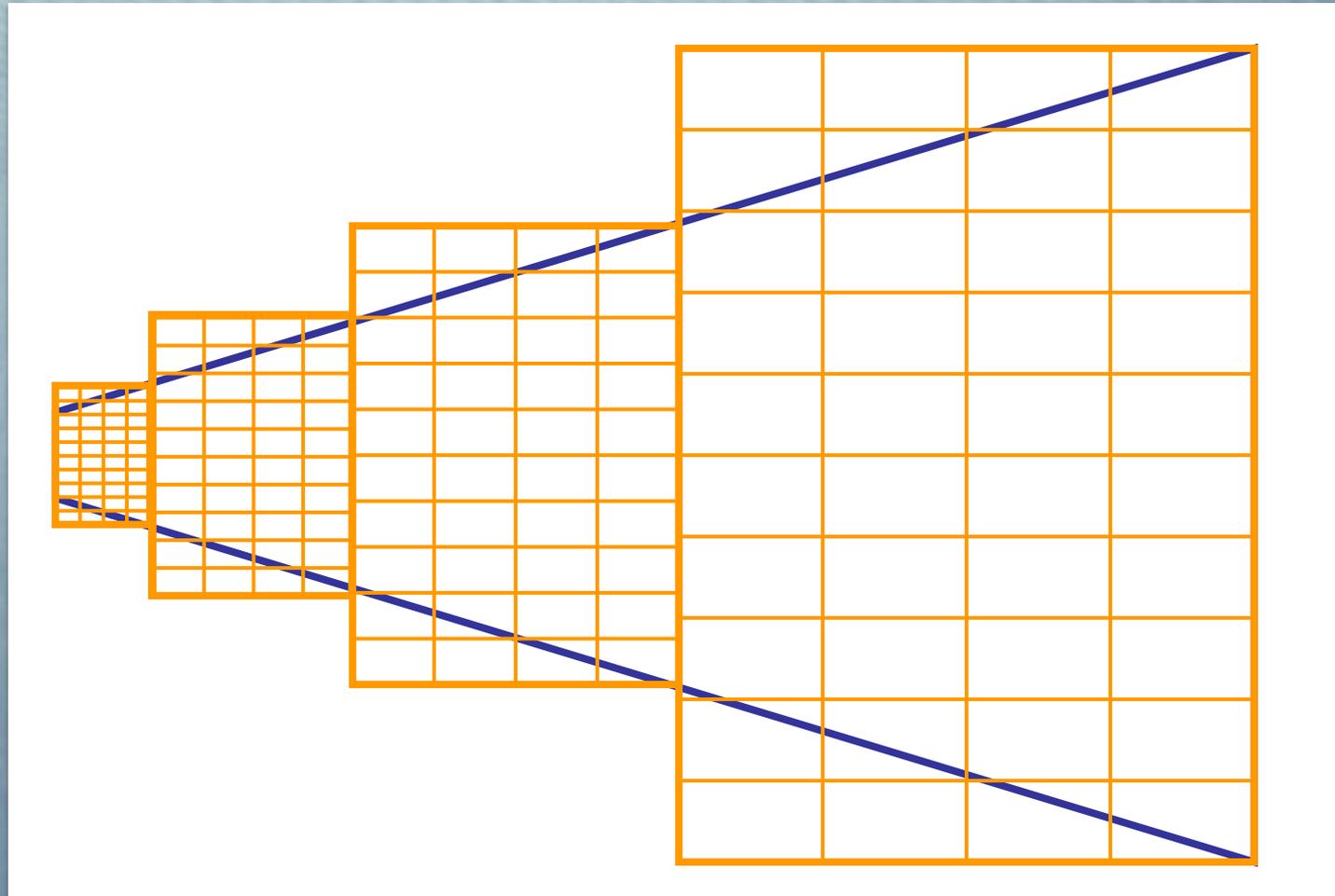
- Caveat: only measured along view direction
- What about $\frac{dp}{dt}$?
 - $\frac{dp}{dt} \sim 1$ for PSM, slightly worse for LiSPSM
- More advanced analysis was done in [Lloyd 2006]
 - Result: “storage factor” constant for n in $[z_n, n_{opt}]$
 - But: best error distribution for n_{opt}



Warping: Problems

- Only works if large z-range visible from light
- Dueling frusta case



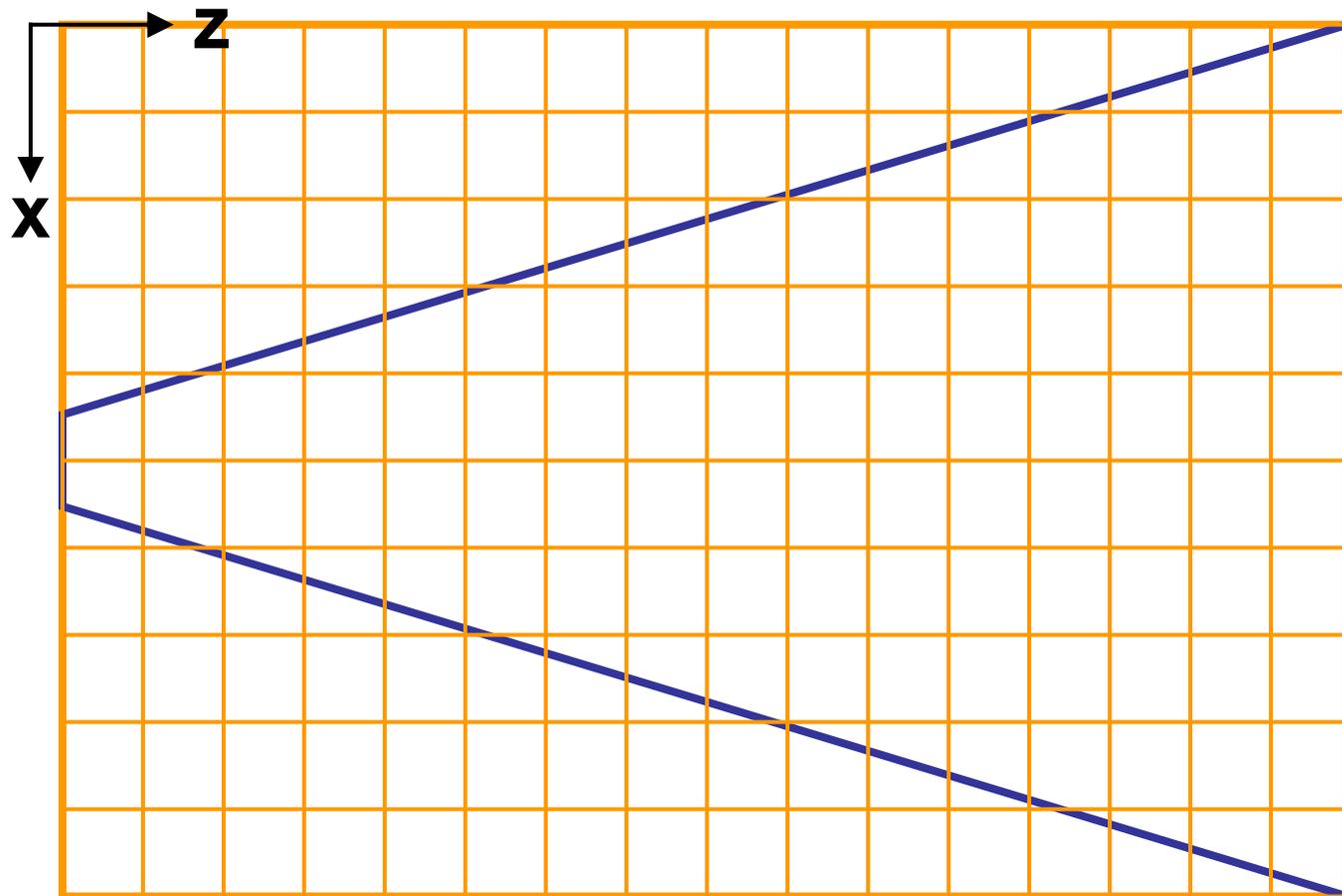


Hard Shadows

Fighting Undersampling – Partitioning

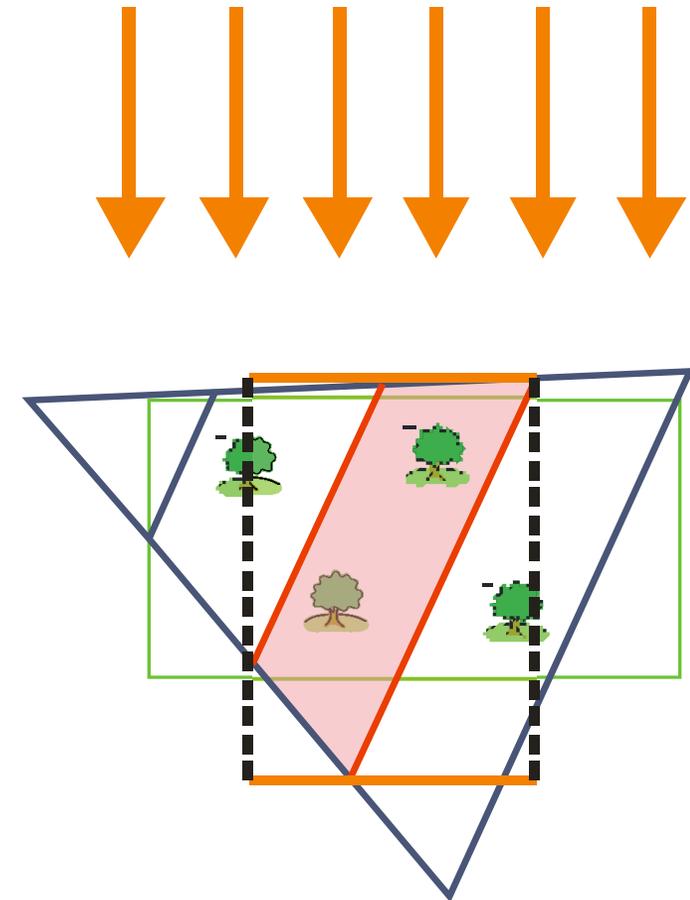
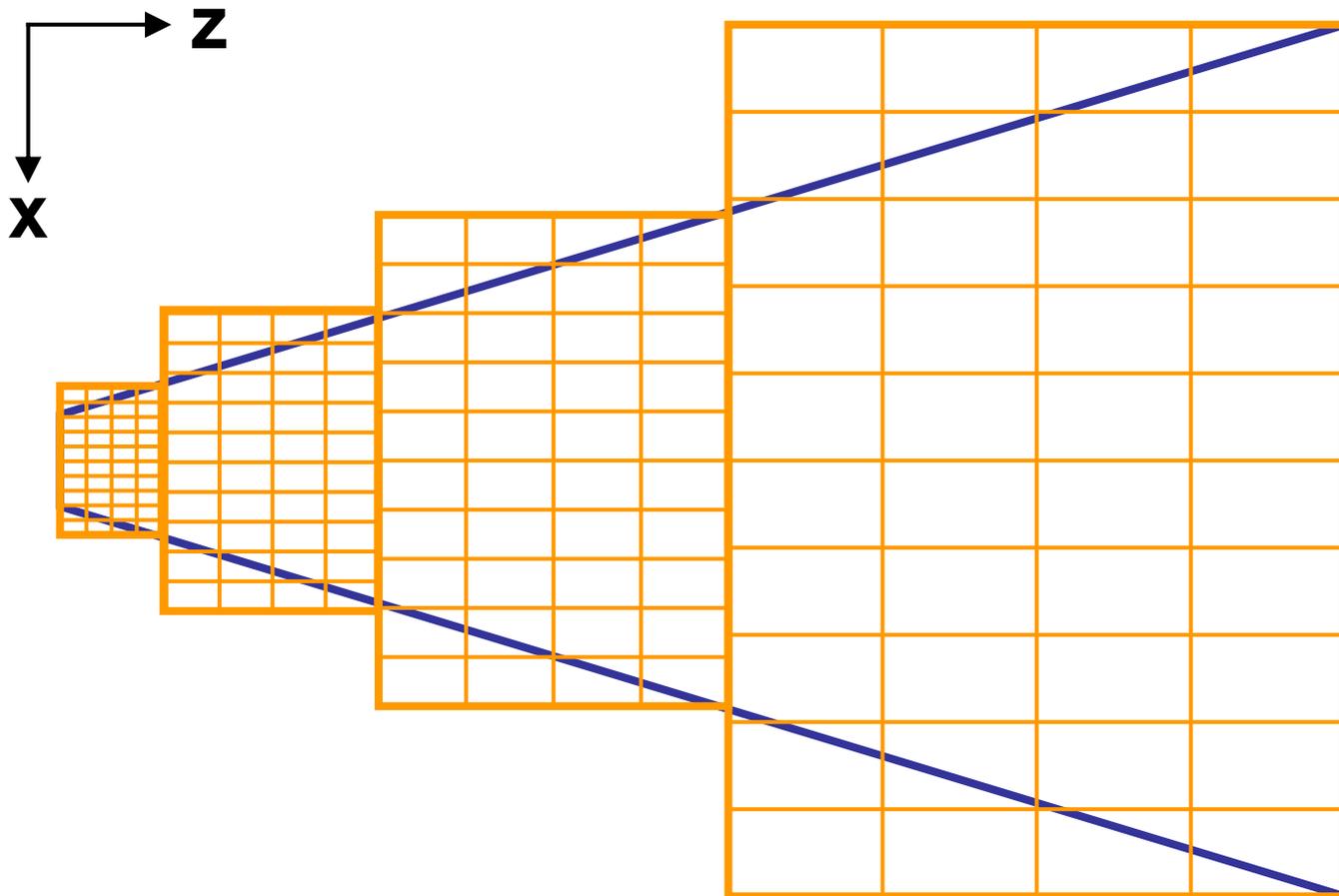
Z-Partitioning: Idea

- Parallel Split Shadow Maps [Zhang 2007]
- Cascaded Shadow Maps [Engel 2007][Zhang2009]



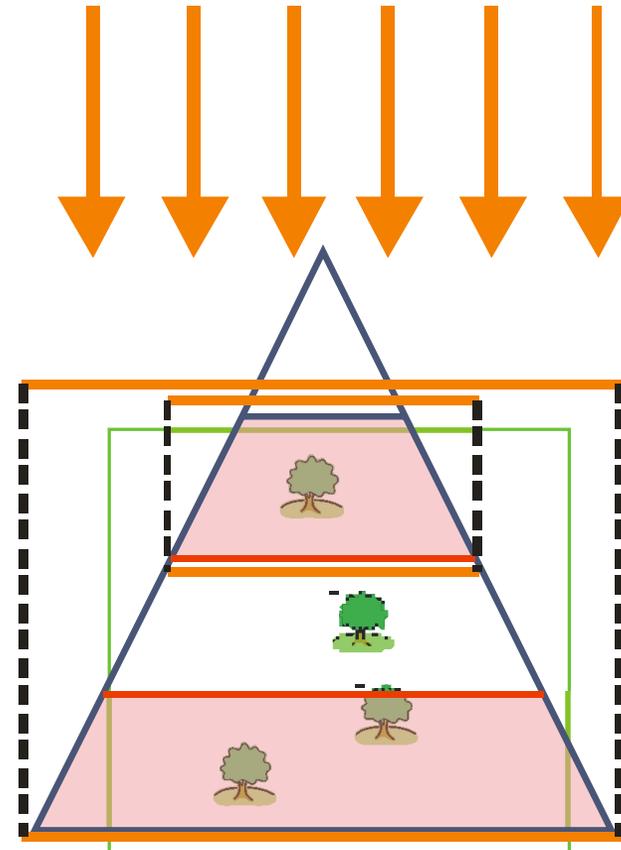
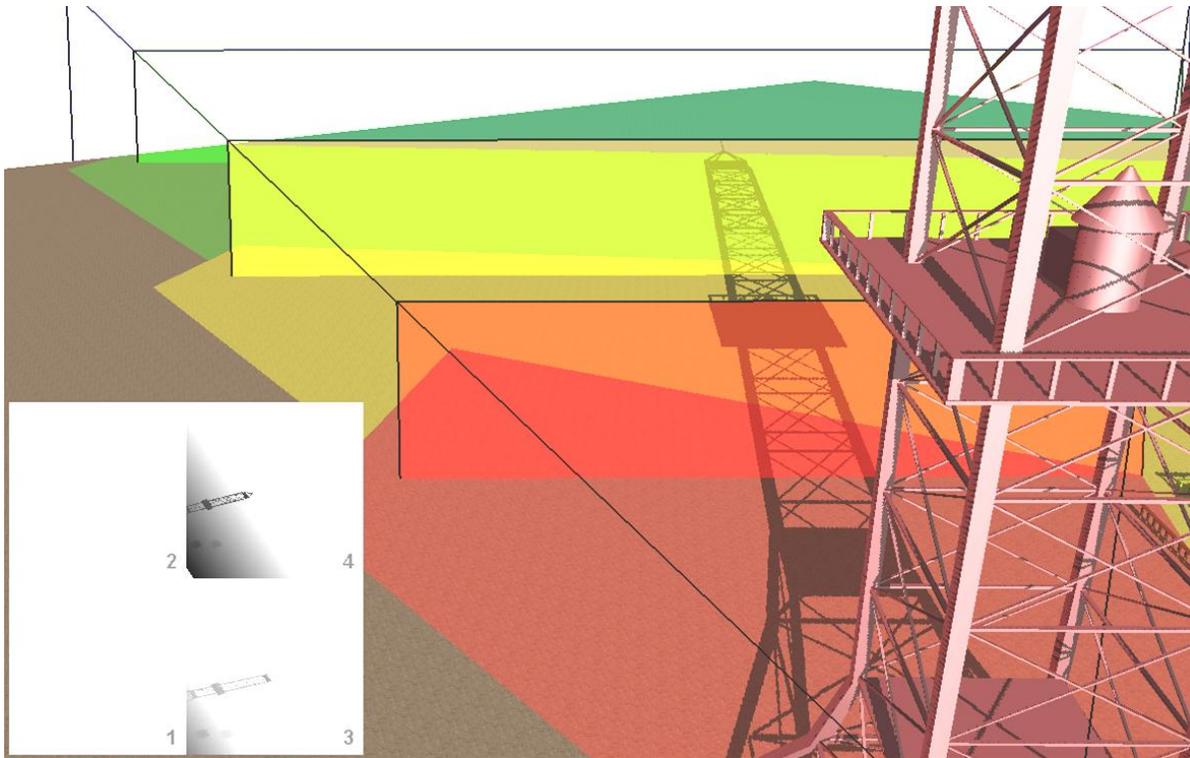
Z-Partitioning

- Partition view frustum into n sub-frusta
- Calculate separate shadow map for each



Z-Partitioning

- Works even in cases where warping fails
 - Light from behind, dueling frusta



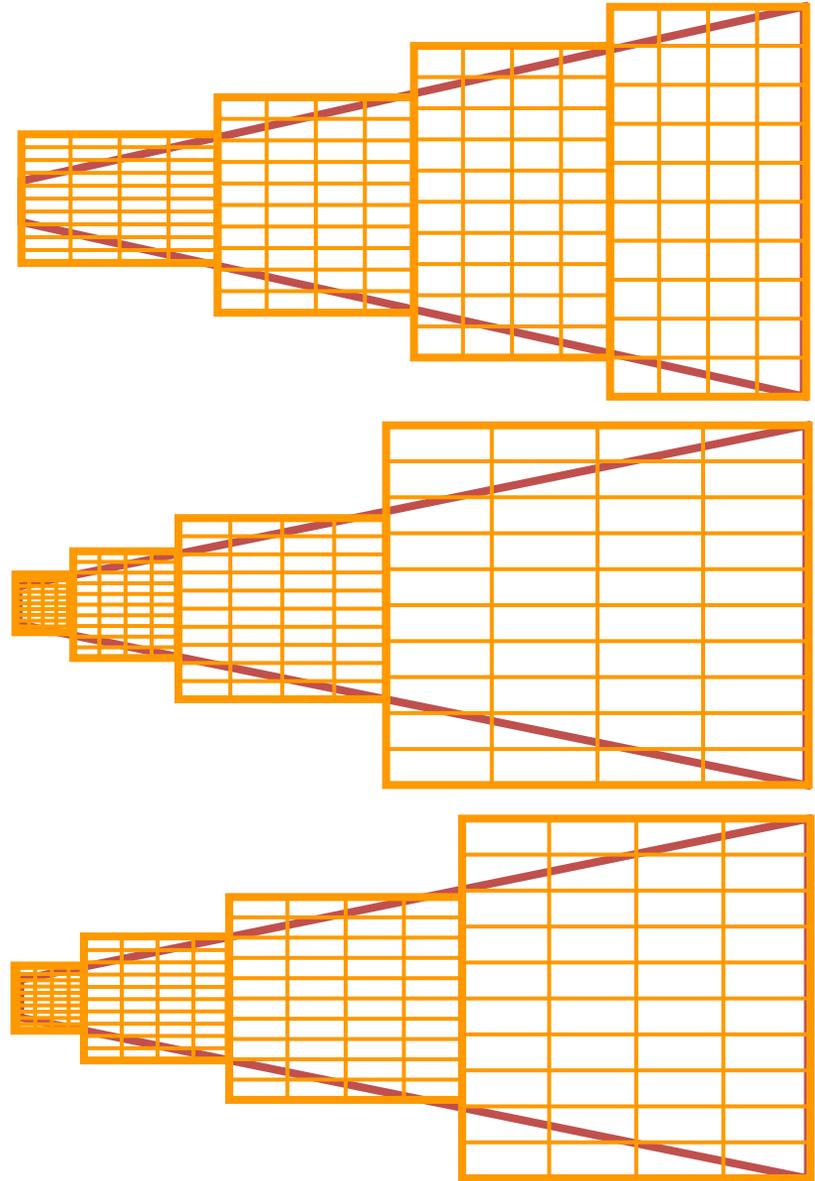
Z-Partitioning

■ How to choose partition sizes?

- Uniform
- Logarithmic/self-similar

$$C_i = z_n \left(\frac{z_f}{z_n} \right)^{\frac{i}{m}}$$

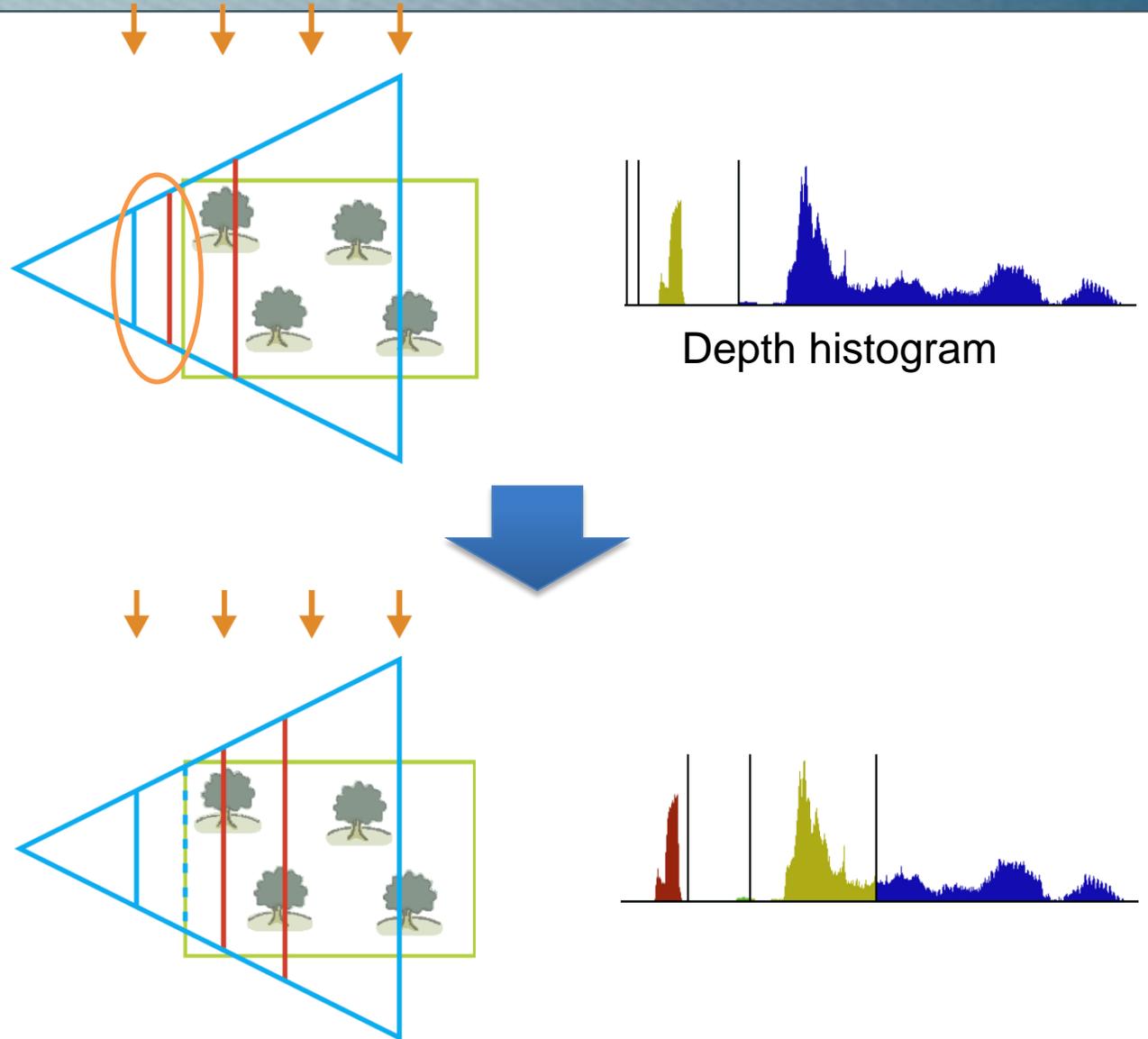
- Linear blend between the two



Z-Partitioning – Sample Distribution Maps

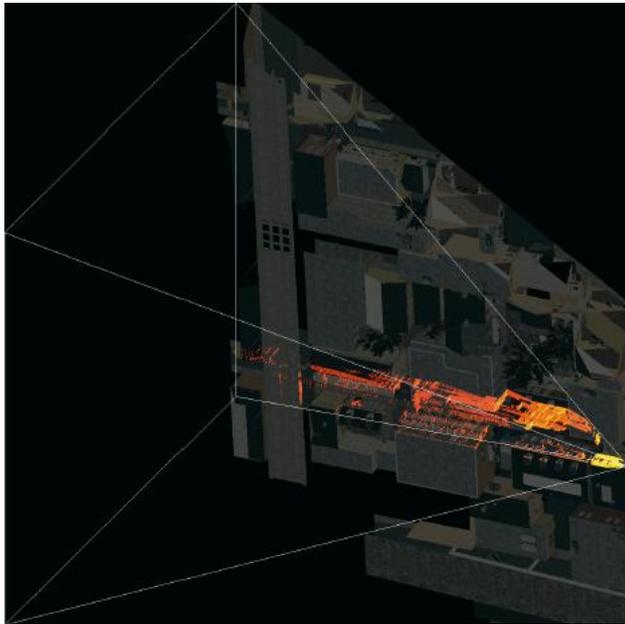
[Lauritzen et al. 2011]

- Optimal partition:
logarithmic
 - Problem: empty areas
- Solution: scene analysis
 - Get min/max-z
 - Adjust depth bounds

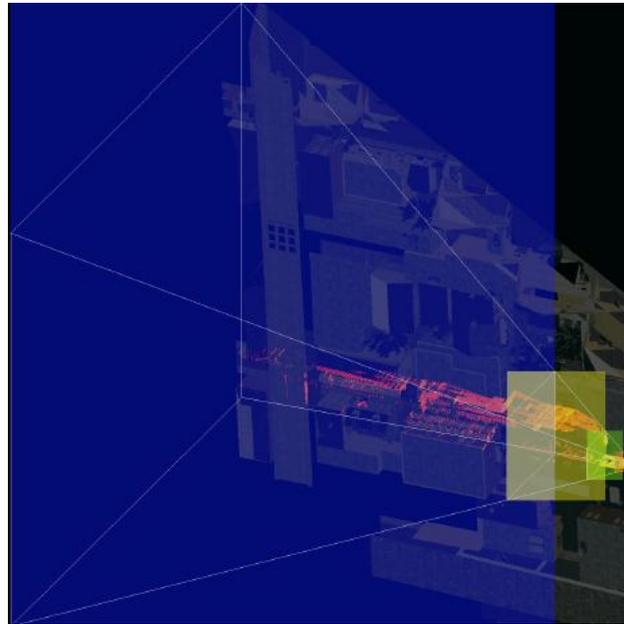


Z-Partitioning – Sample Distribution Maps [Lauritzen et al. 2011]

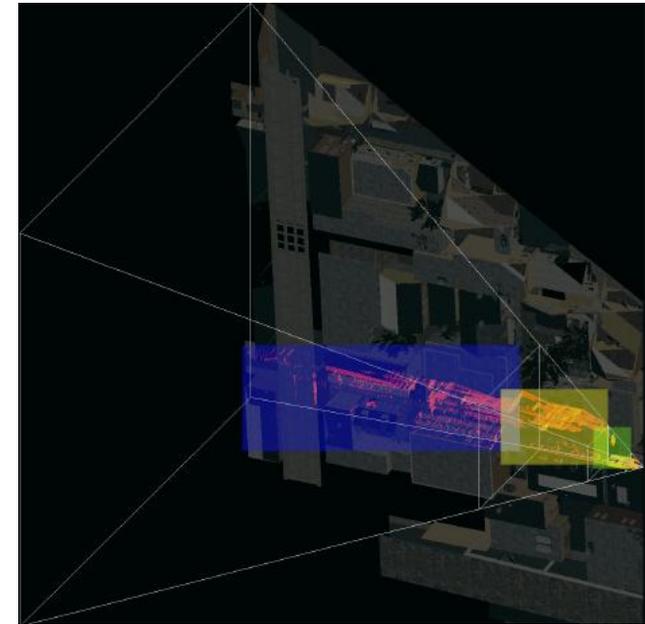
- Extension: analyze fragment distribution in light space
 - Tightly fit partitions in s and t



Shadow map



Standard Z-partitions

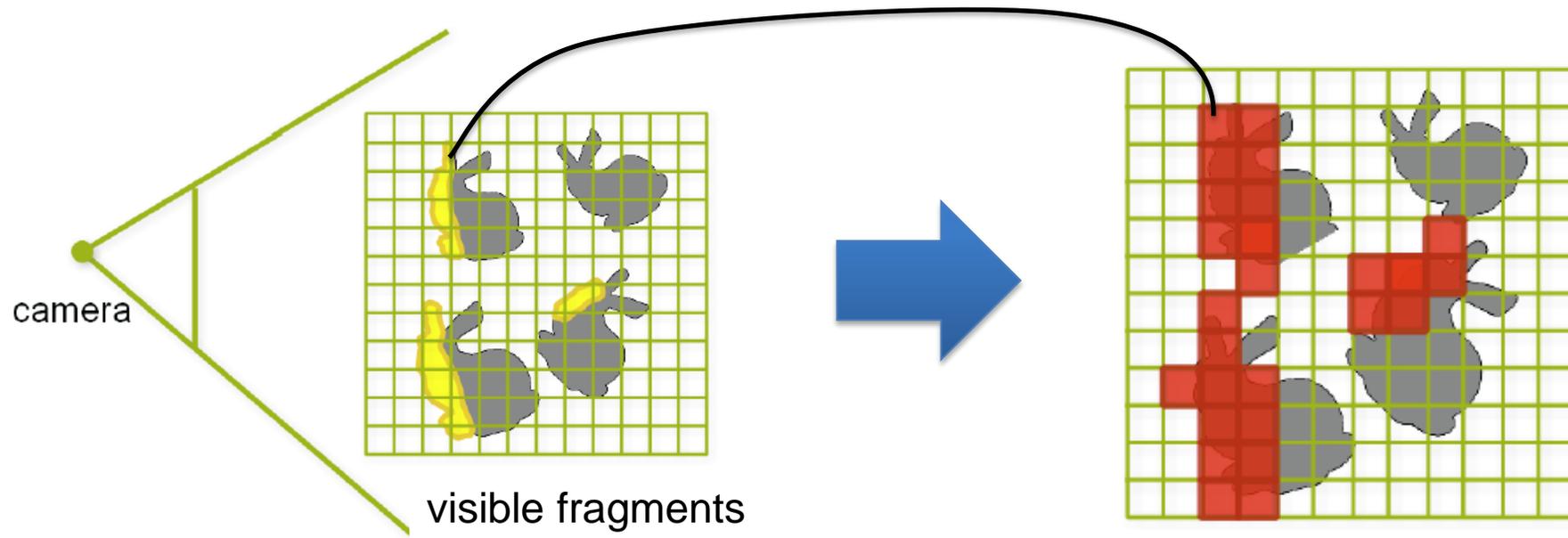


Bounded partitions

Shadow Caster Culling

[Mattausch et al. 2011]

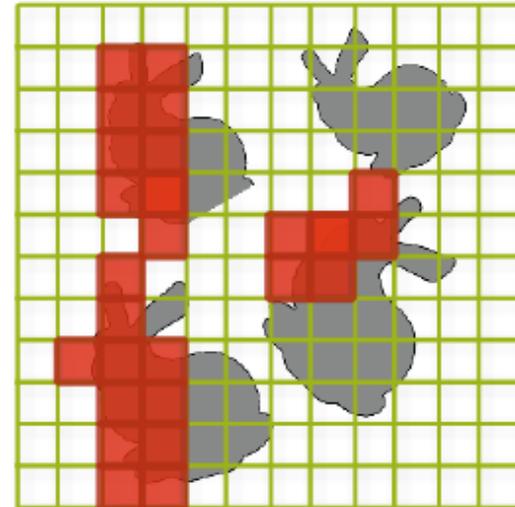
- How to determine relevant fragments in light space?
 - Determine shadow receivers (camera pass)
 - Render shadow receivers into light-space mask (light pass)
 - Fragment-level check using reverse shadow lookup



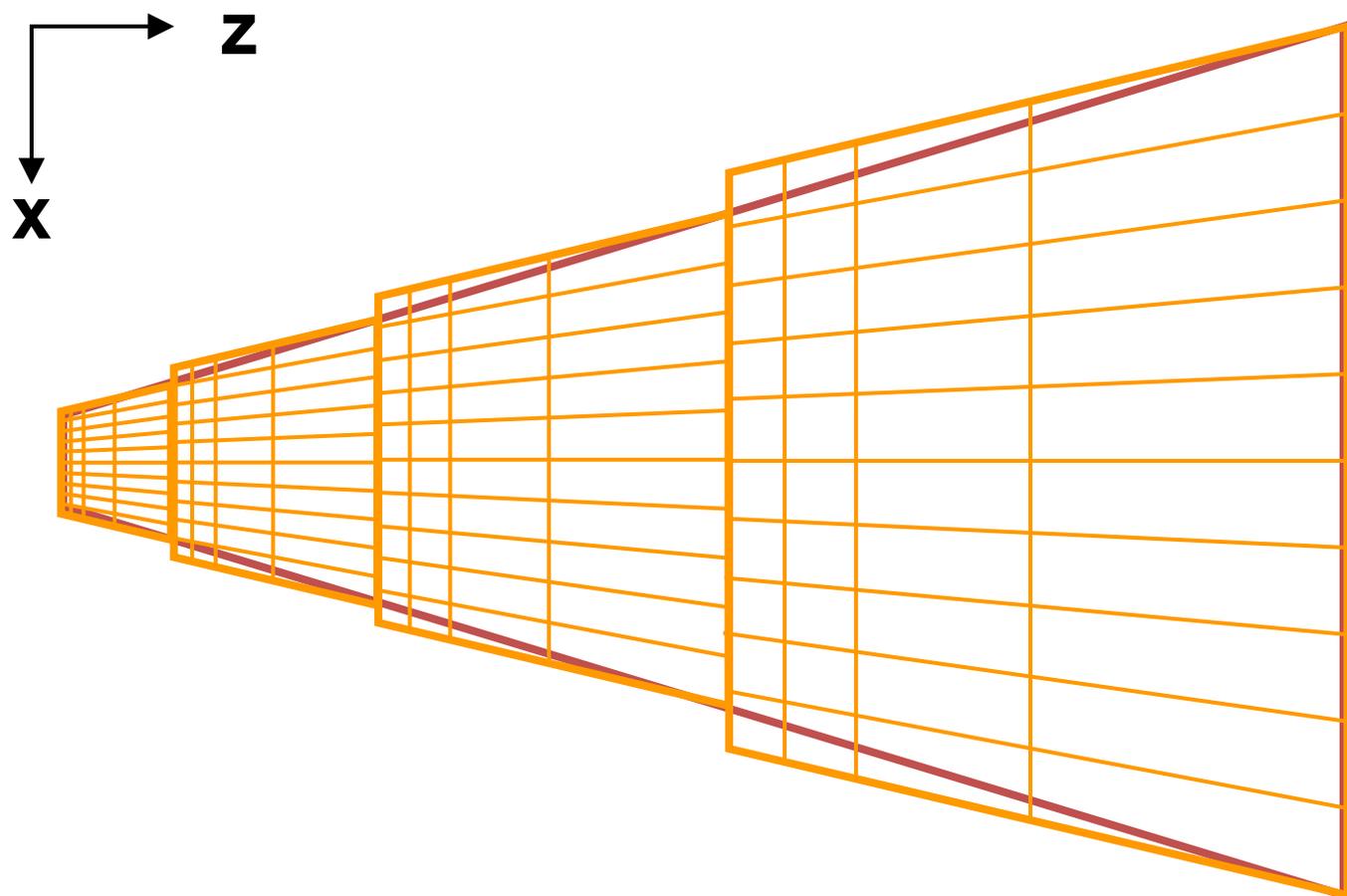
Shadow Caster Culling

[Mattausch et al. 2011]

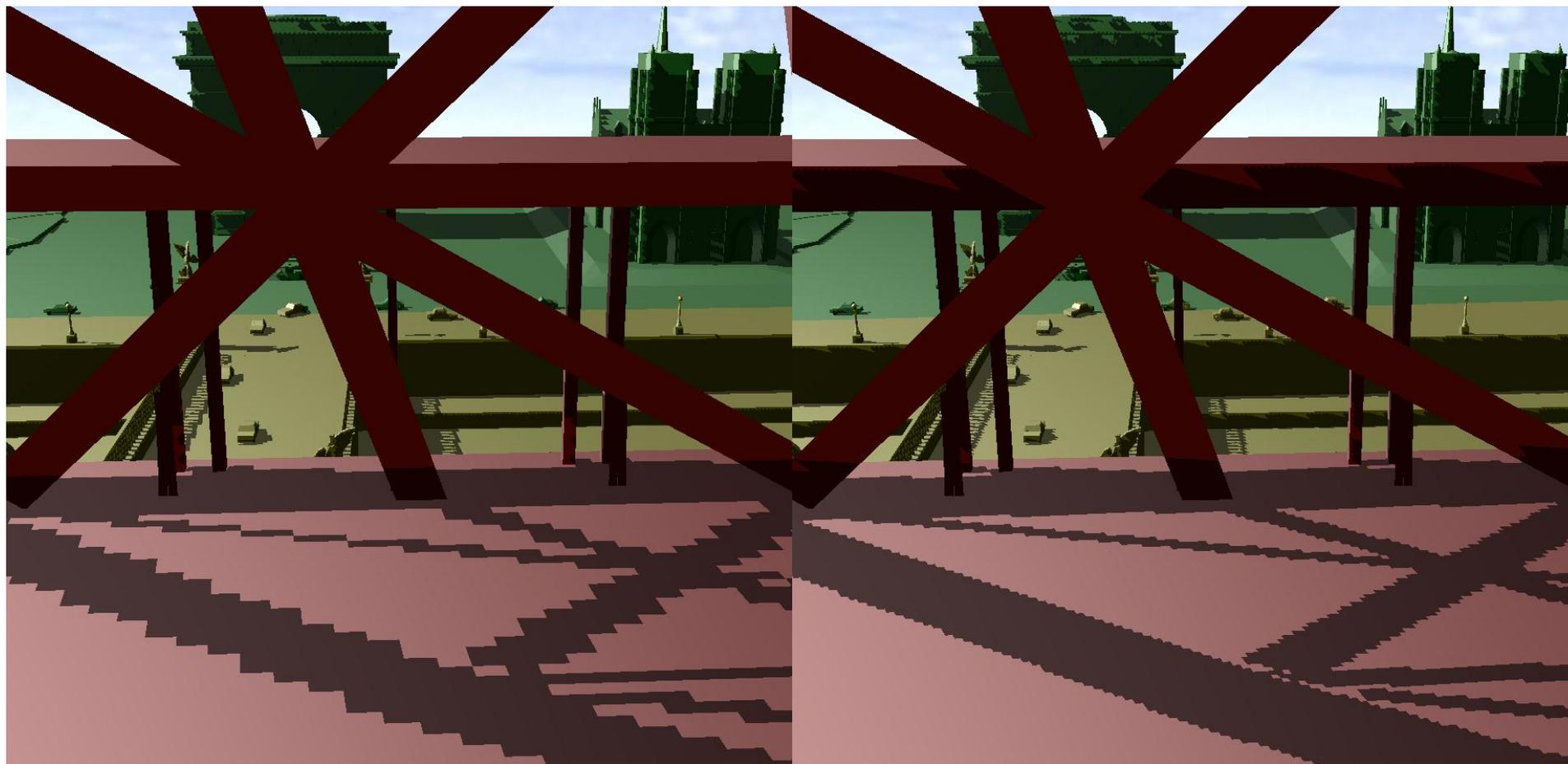
- Culling:
 - Use mask for occlusion culling for depth map
 - Large performance gain for outdoor scenes!



Z-Partitioning and Warping



Z-Partitioning and Warping

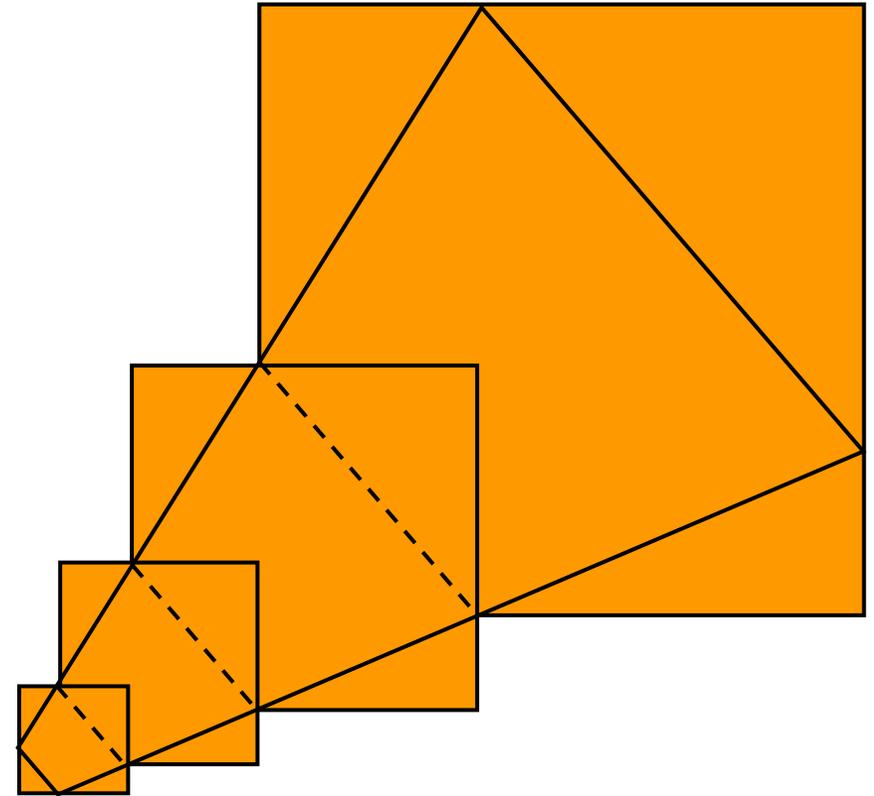


Partitioning

Partitioning + warping

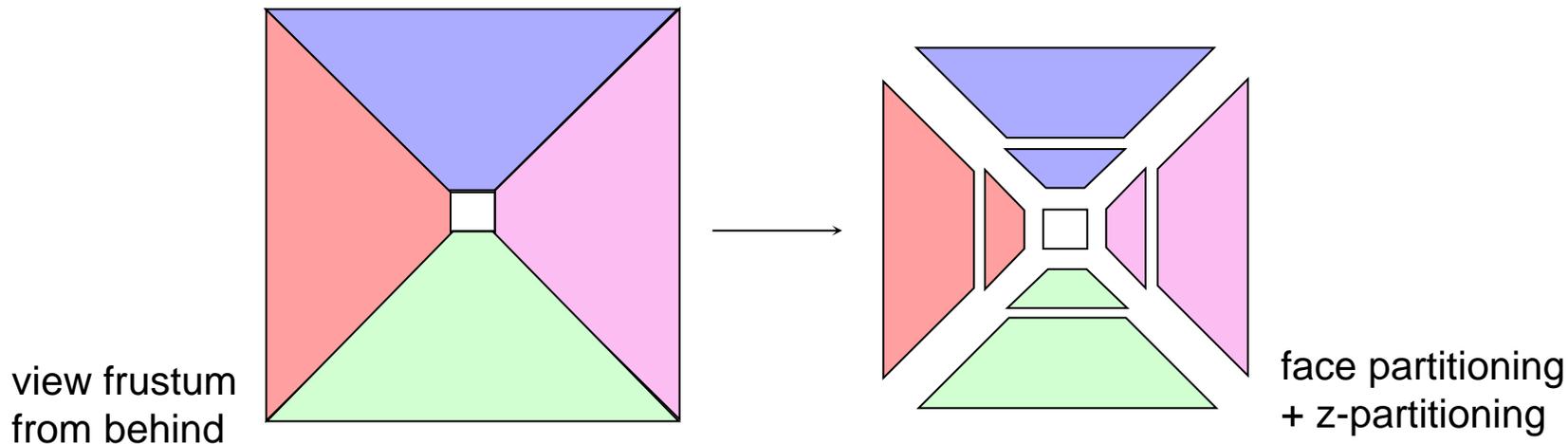
Temporal coherence

- Fix coordinate system orientation in world space
- Shadow maps move at integral multiples of a texel width (see fitting)
- Gives up warping

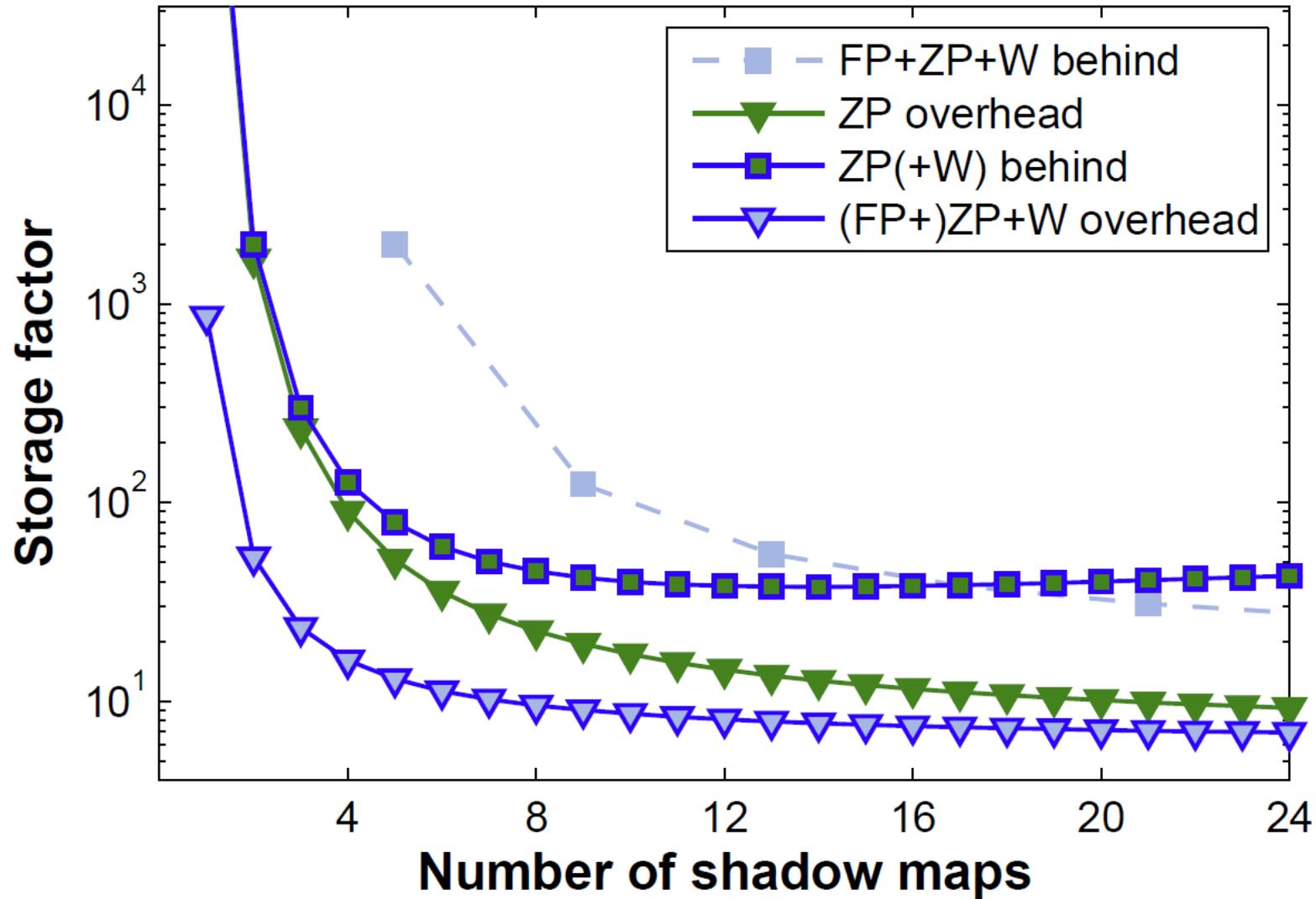


Face Partitioning [Lloyd et al. 2006]

- Alternative to z-partitioning
 - Partition frustum according to faces
- Can be combined with z-partitioning (see later)
 - Reduces redundancy
 - Also works without warping (“oblique projection”)

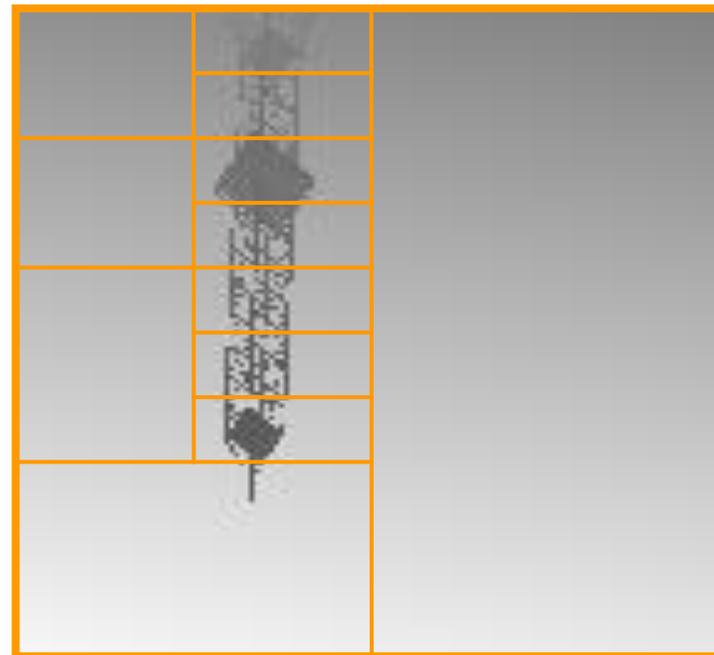


Full Error Analysis [Lloyd et al. 2006]



Adaptive Partitioning

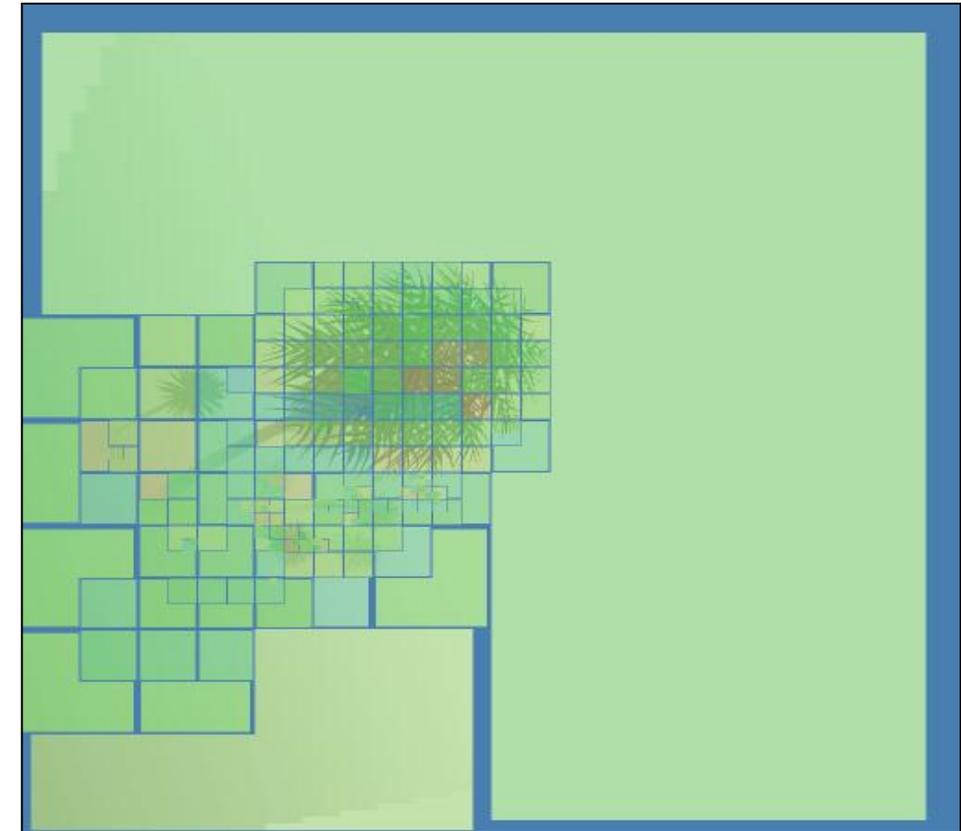
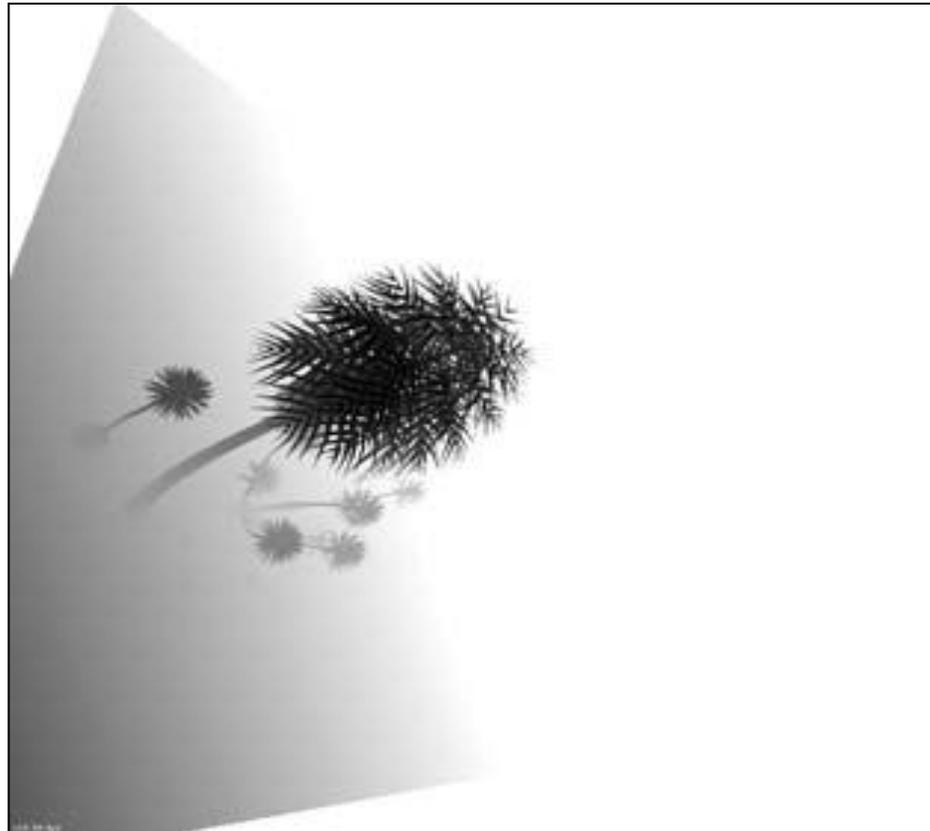
- Warping and z-partitioning are **global** resampling schemes
 - Deal with perspective aliasing
 - Projection aliasing needs local scene adaptive resampling!
- Adaptive partitioning adaptively splits shadow map
 - Usually quad-tree subdivision
 - Algorithms mainly differ in termination criteria



Adaptive Shadow Maps

[Fernando et al. 2001; Lefohn et al. 2006]

- High resolution only needed at edges
- Search for edge (slow)
 - If edge → split



Fitted Virtual Shadow Maps/Resolution Matched SM

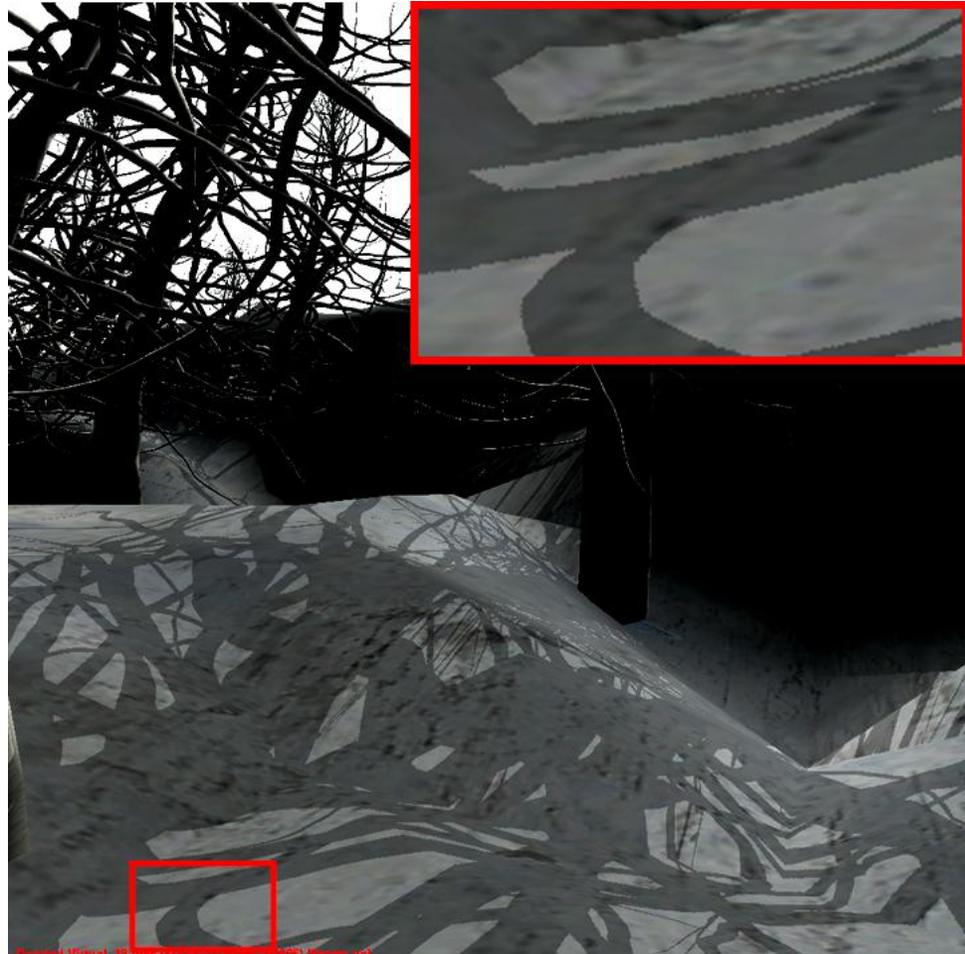
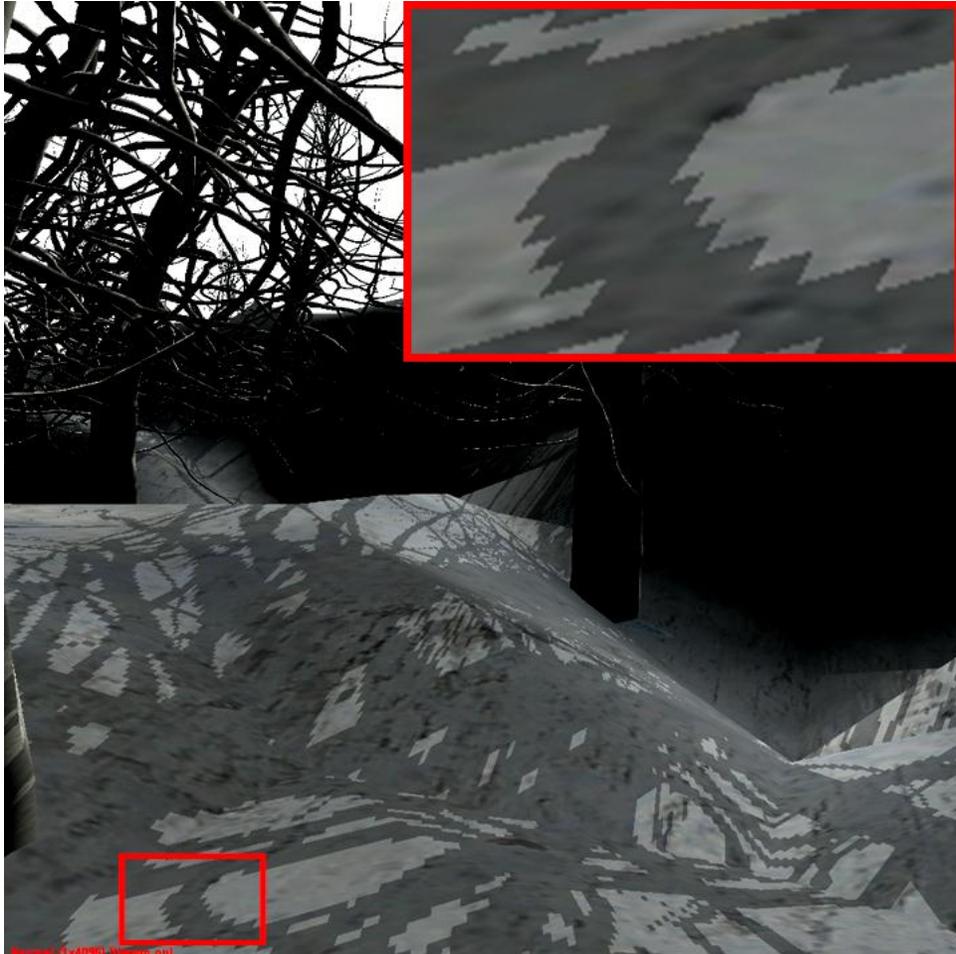
[Giegl & Wimmer 2007], [Lefohn et al. 2007]



- Do not calculate all quad-tree levels, but determine finest levels necessary
 - Camera prepass
 - Analyzed on CPU (FVSM) or GPU (RMSM)

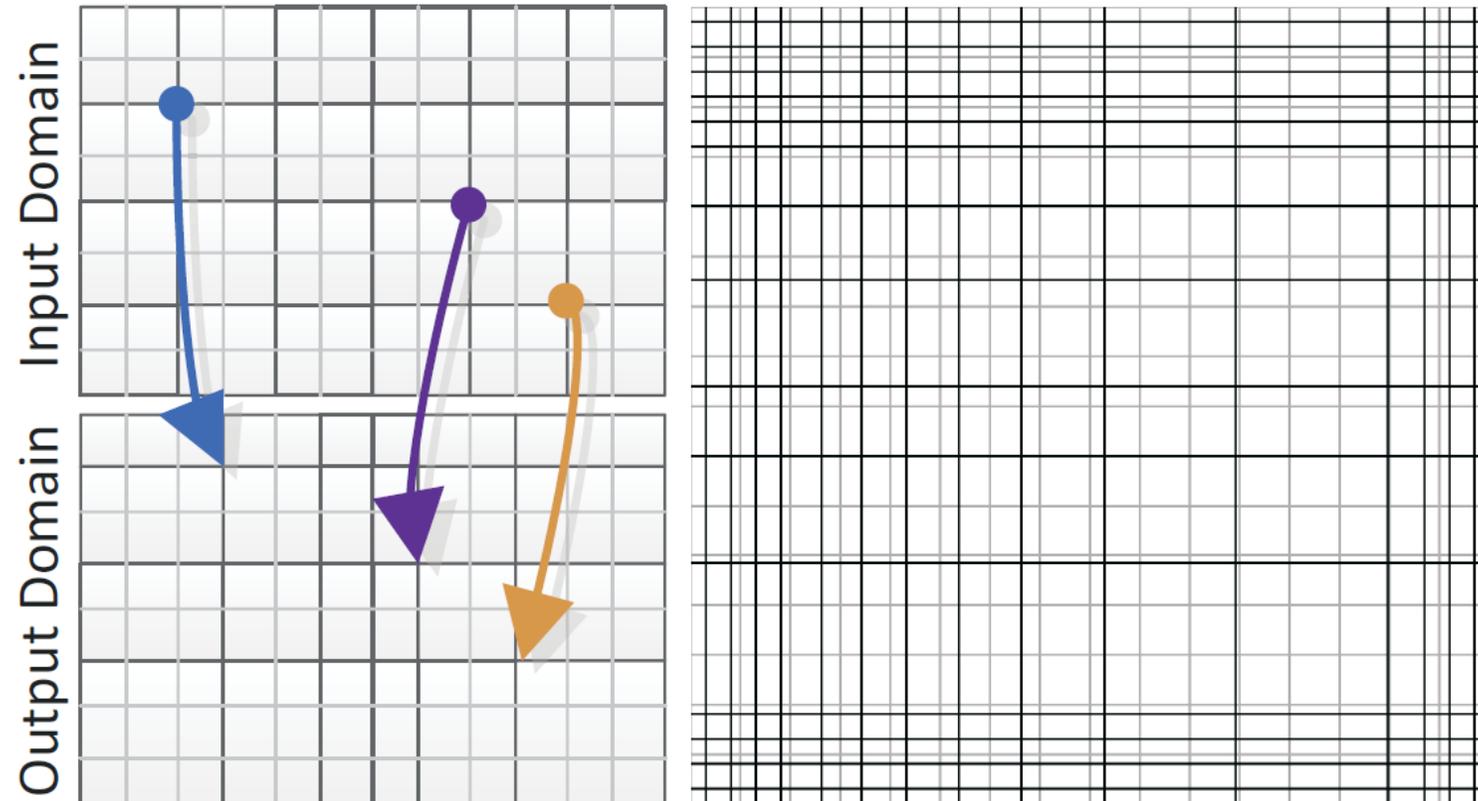
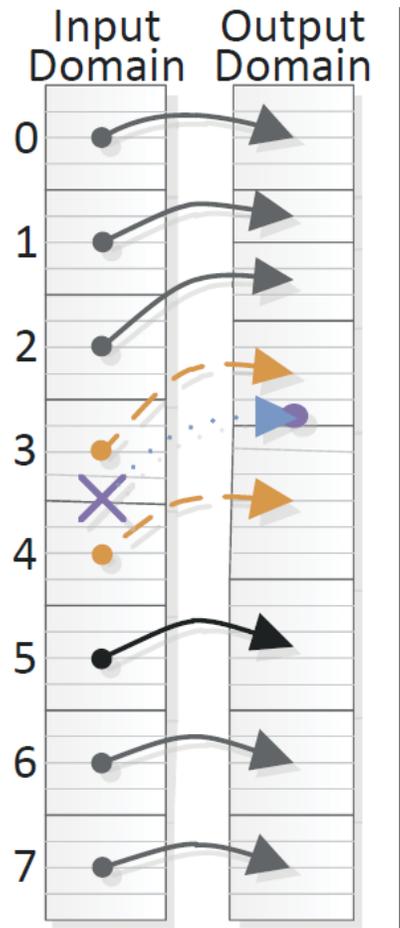
Fitted Virtual Shadow Maps

[Giegl & Wimmer 2007]



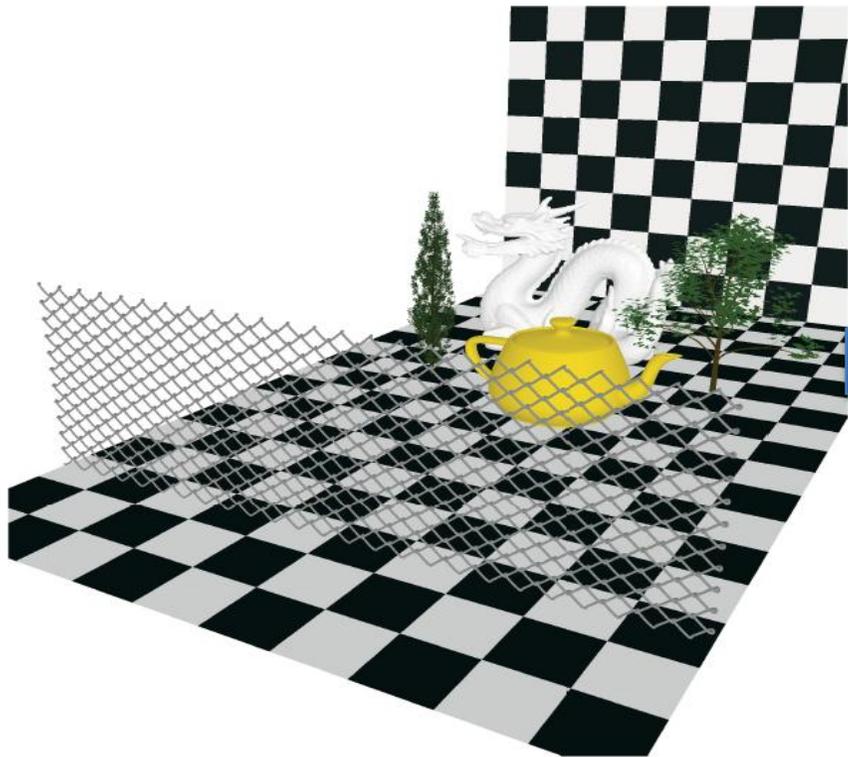
Rectilinear Warping [Rosen et al. 2012]

- Apply 2 separate 1D-warps based on scene content

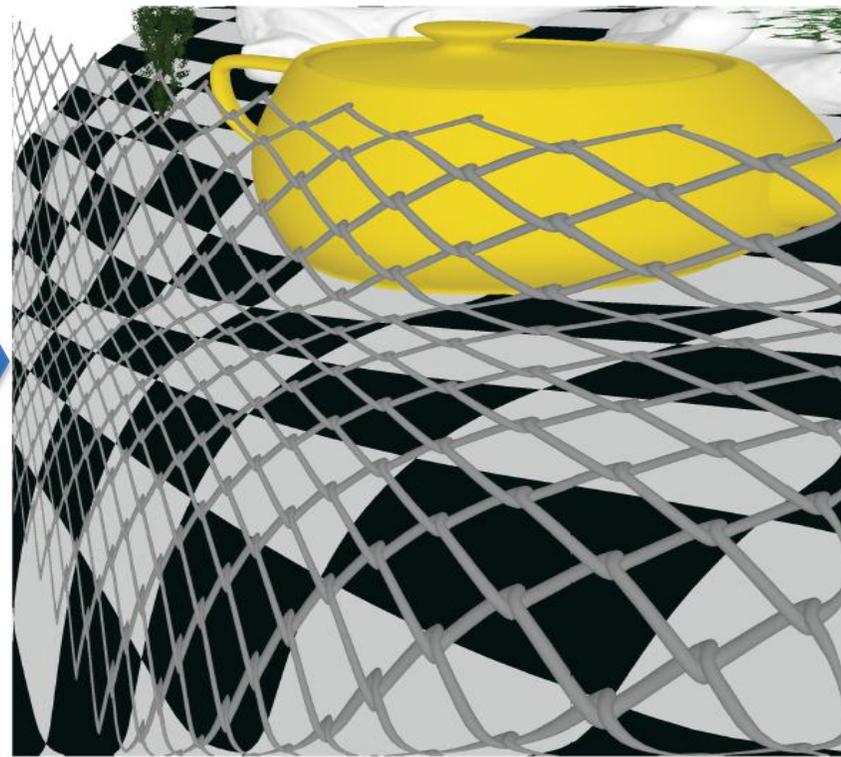


Rectilinear Warping [Rosen et al. 2012]

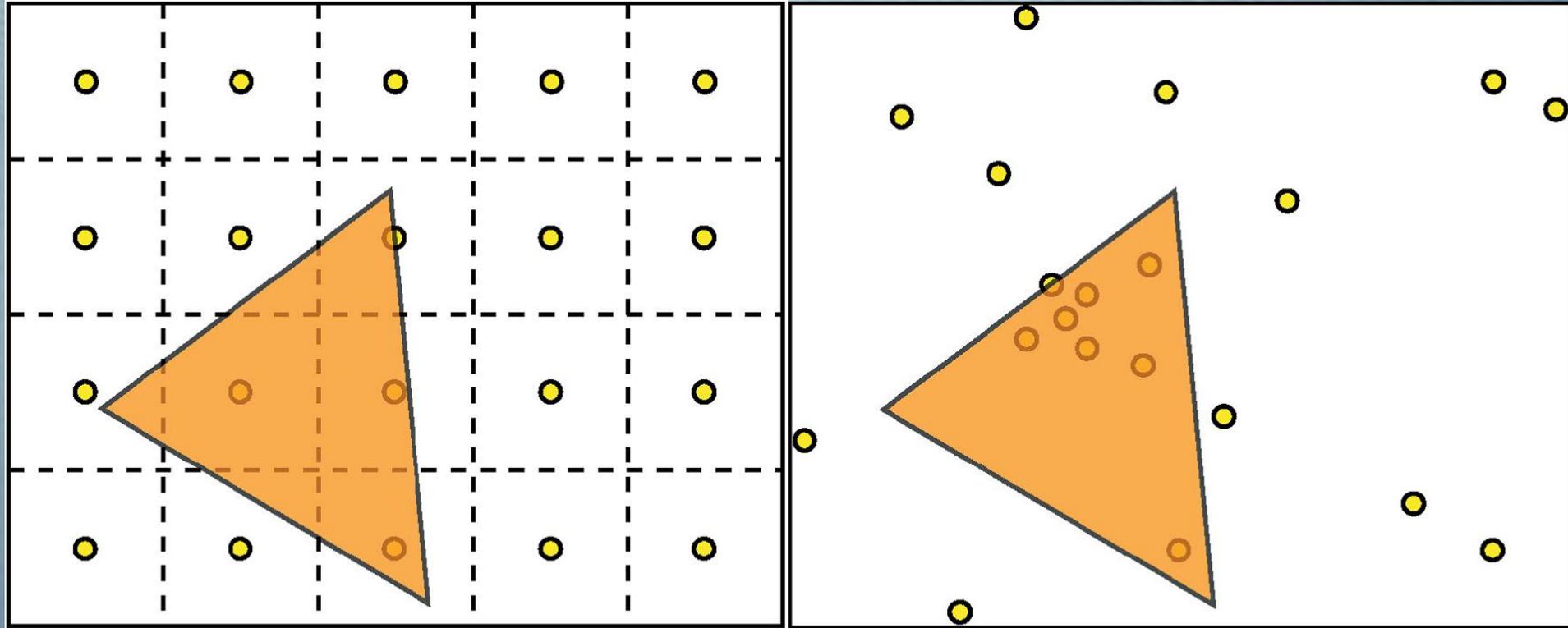
- Requires scene analysis pass
- Requires tessellation to create shadow map



Standard shadow map



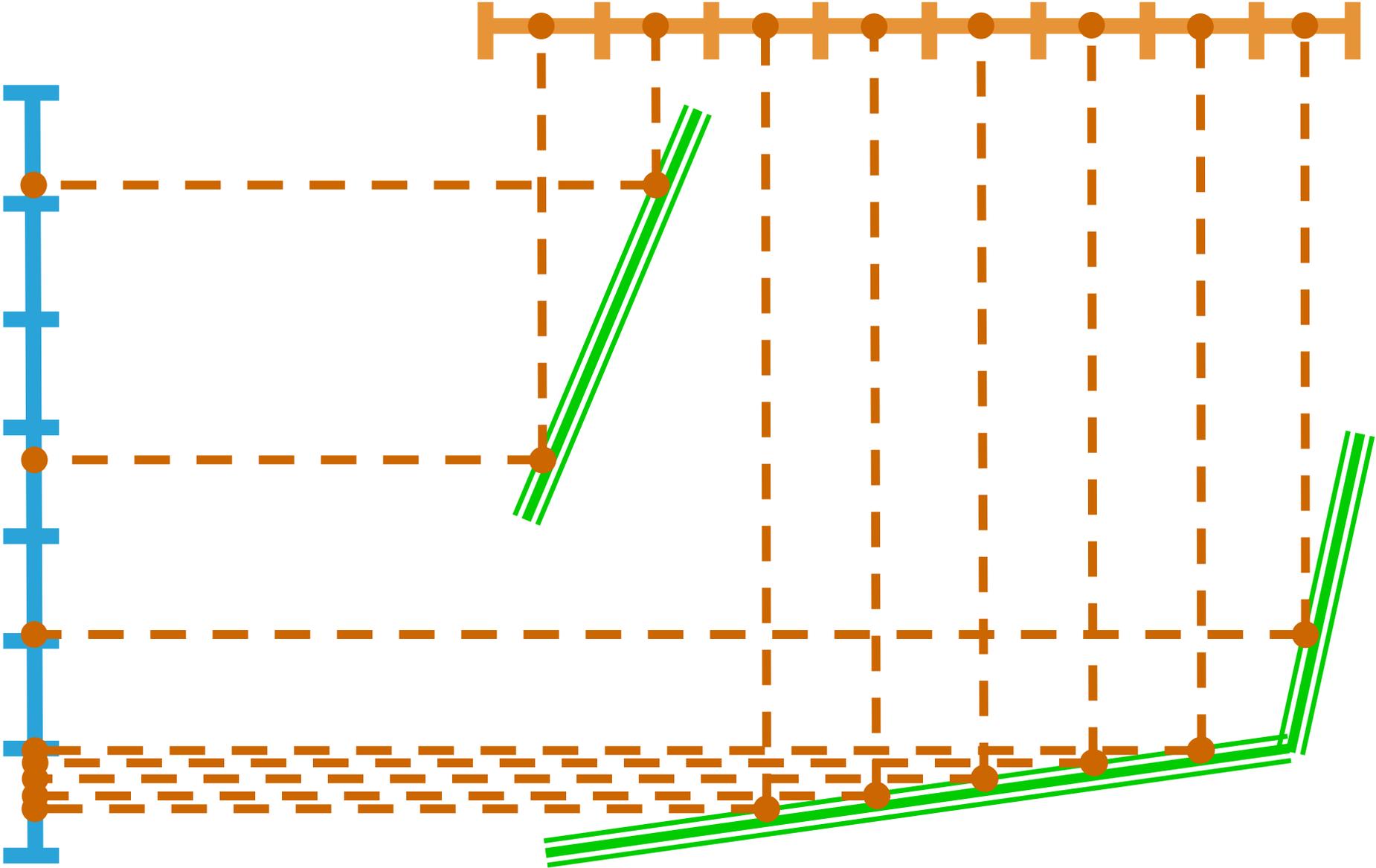
Rectilinearly warped shadow map



Hard Shadows

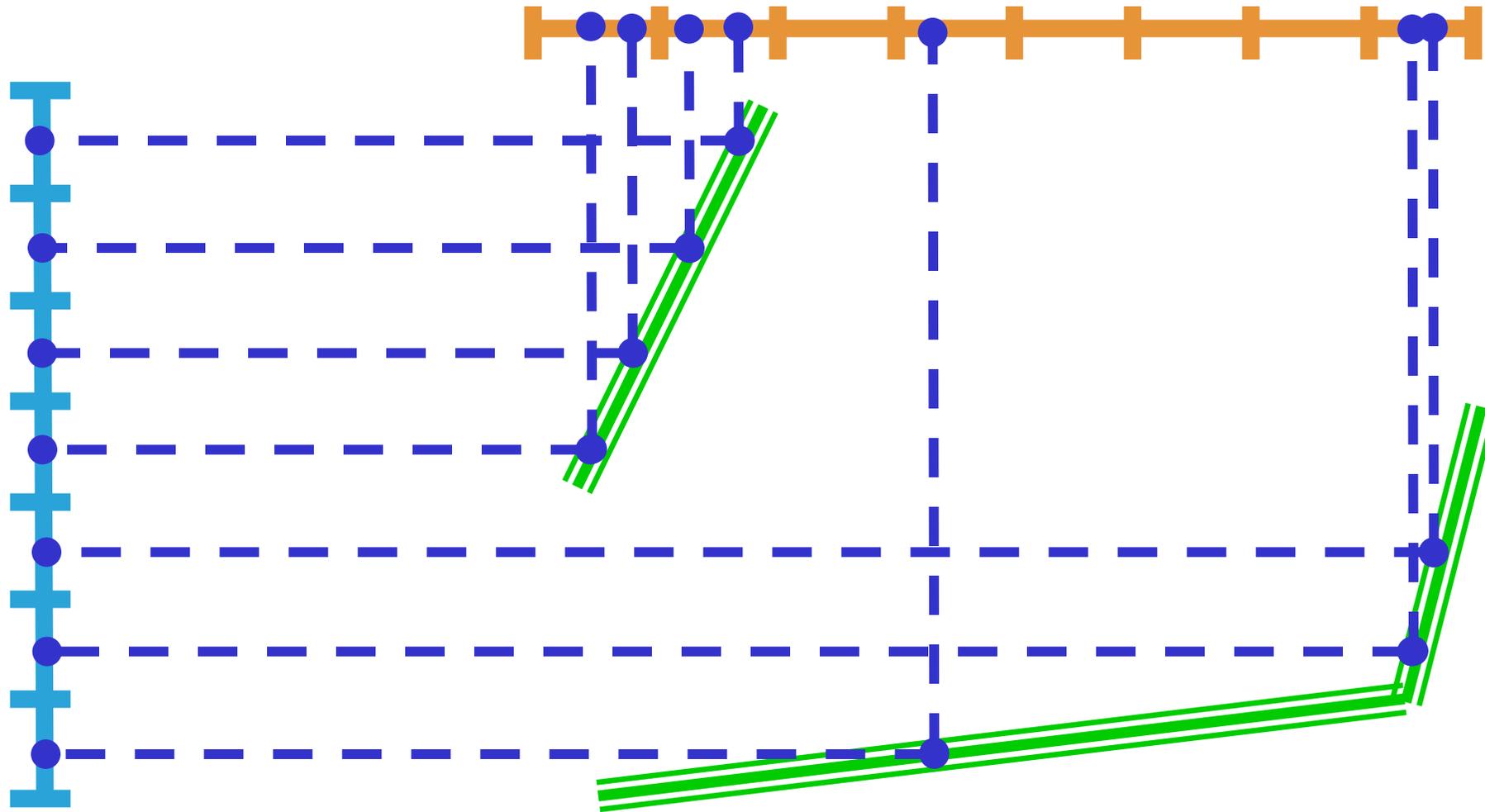
Fighting Undersampling – Irregular Sampling

Shadow Mapping Sampling



What Samples Do We Want?

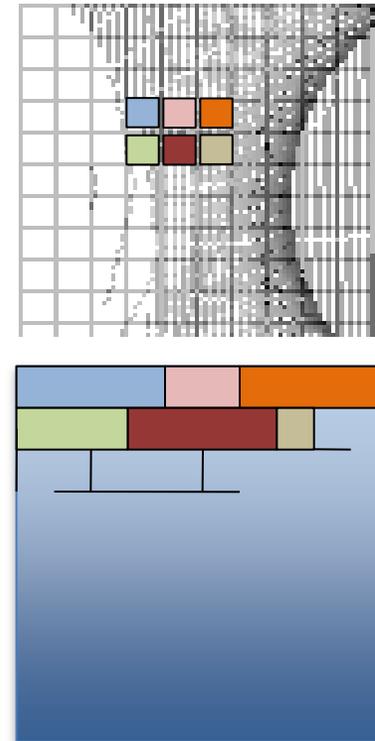
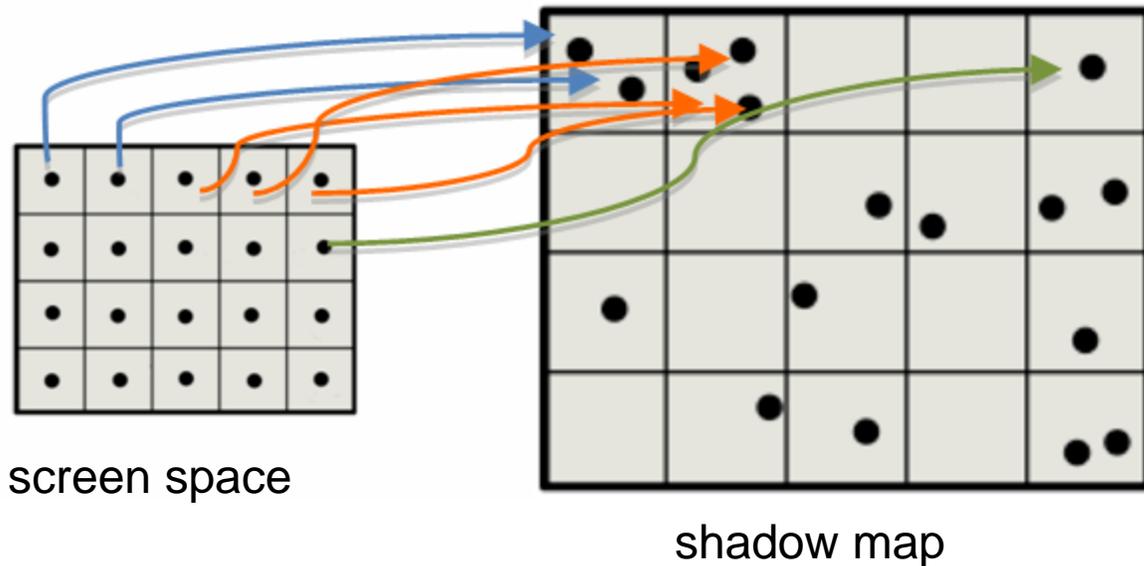
- Idea: use eye space samples to generate shadow map samples



Alias Free Hard Shadows

[Aila and Laine 2004][Sintorn et al. 2008]

- Camera pass: transform and project view-samples into light-space
 - Store in a compact data structure with a list per light-space texel

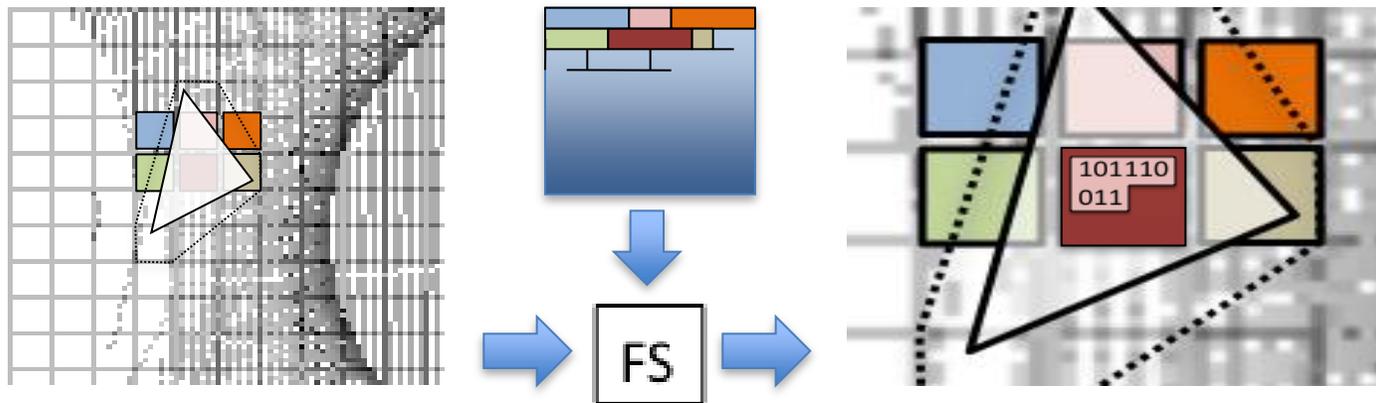


Alias Free Hard Shadows

[Sintorn et al. 2008]



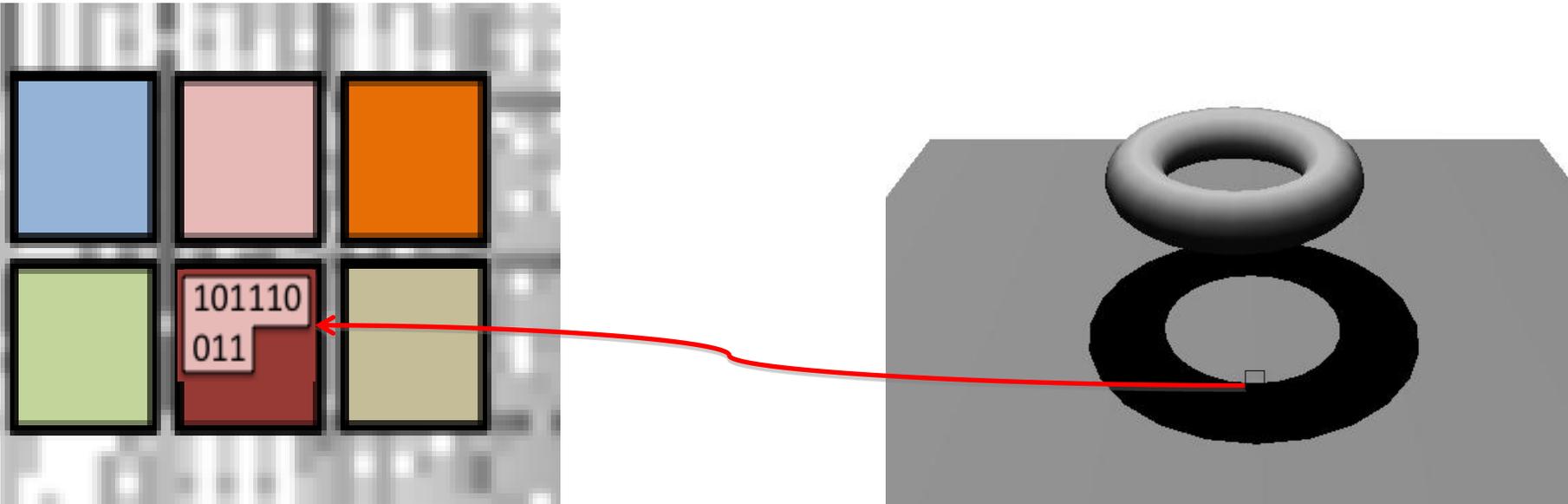
- Light pass: render all geometry (conservatively)
 - For each generated fragment, test all view-samples in list against triangle
 - Set corresponding output bit depending on occlusion



Alias Free Hard Shadows

[Sintorn et al. 2008]

- Final screen-space pass
- Use bitmask from previous pass for shadowing

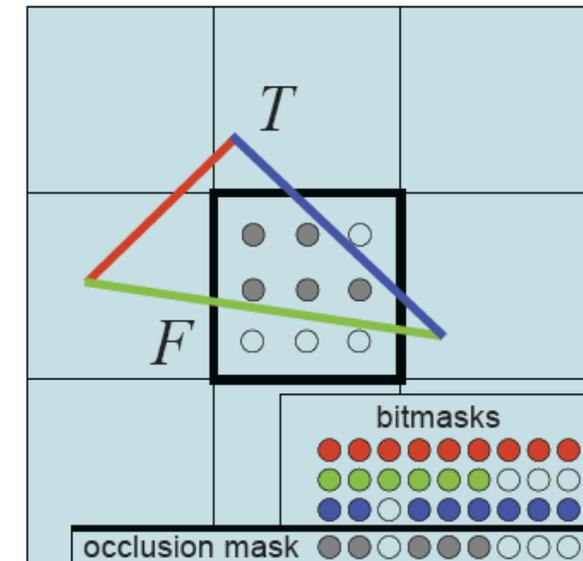
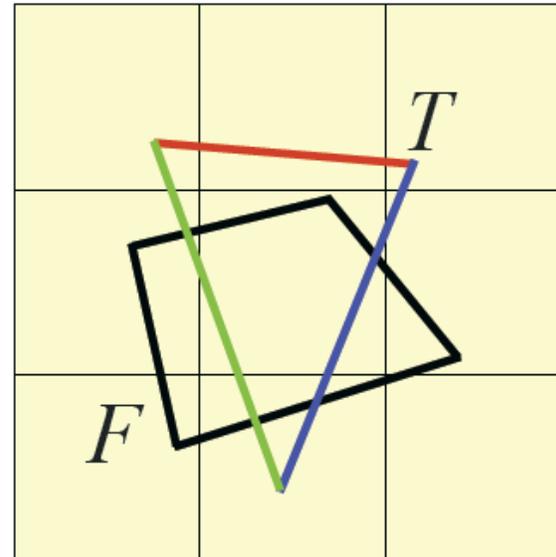
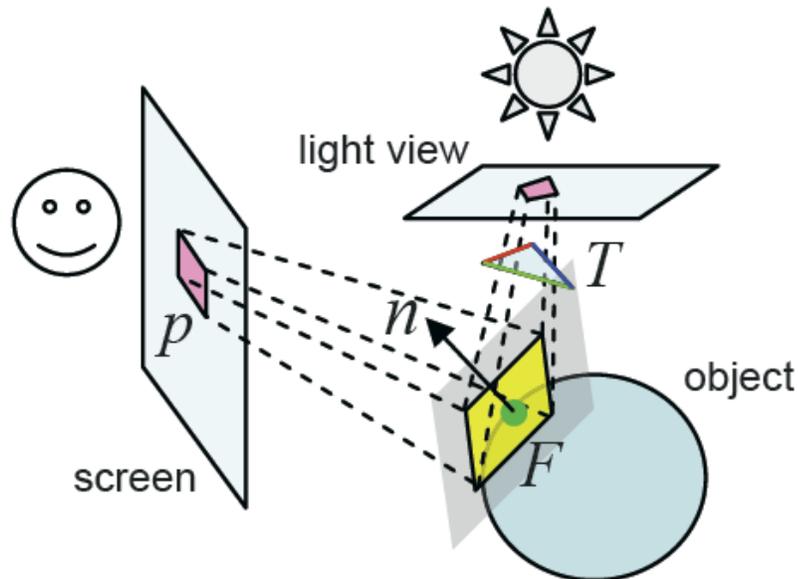


Subpixel Alias Free Shadow Maps

[Zhou et al. 2009]



- Problem: no antialiasing (supersampling)
- Solution:
 - Project facets instead of samples
 - Precompute subsample tests (constant time [Eisemann 2007])



$$\mathit{cache}(n) := \mathit{conf} * s(n) + (1 - \mathit{conf}) * \mathit{cache}(n-1)$$

Hard Shadows

Temporal Reprojection

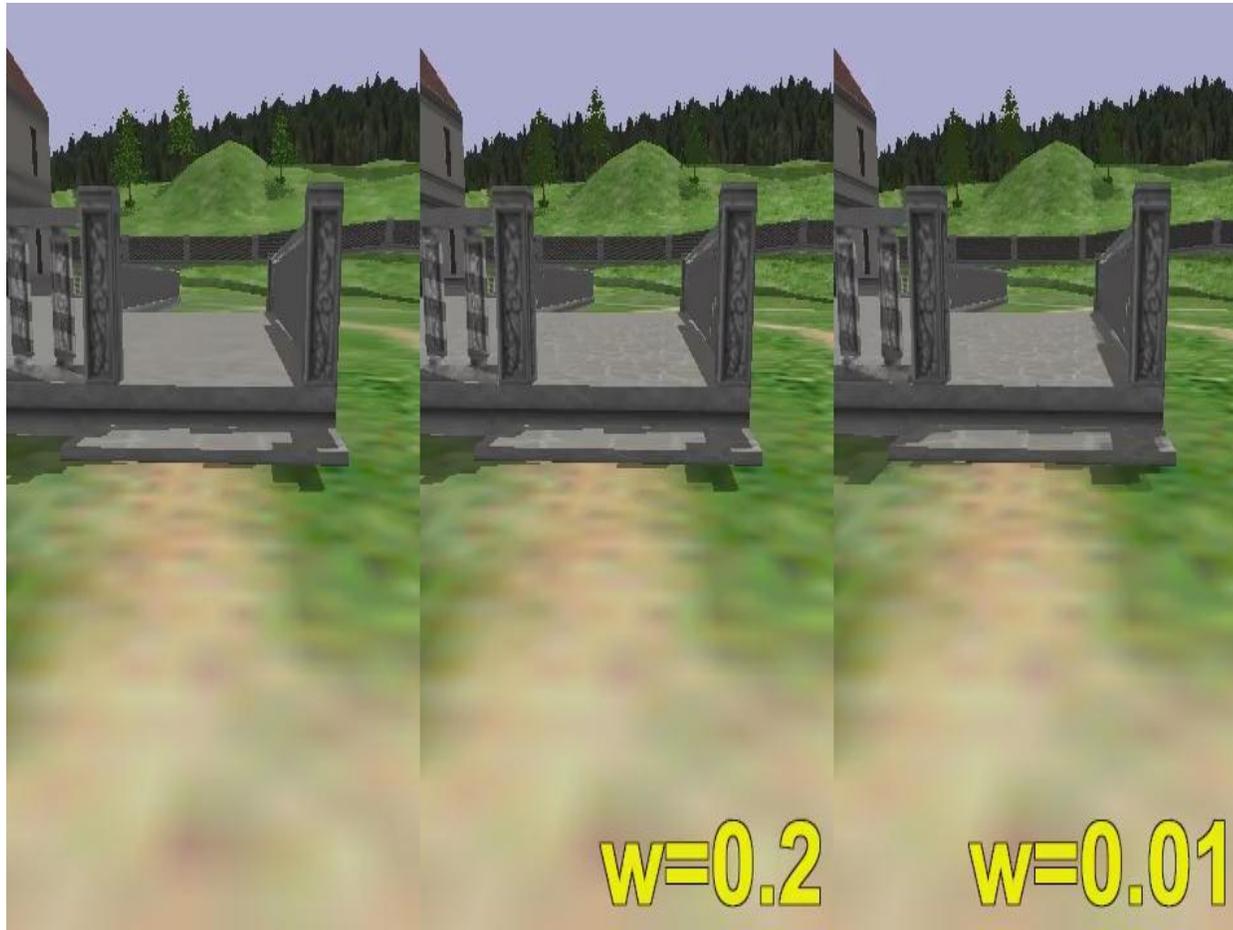
Temporal Smoothing

- Shadowing result of previous frame ($n-1$) is stored in *cache*

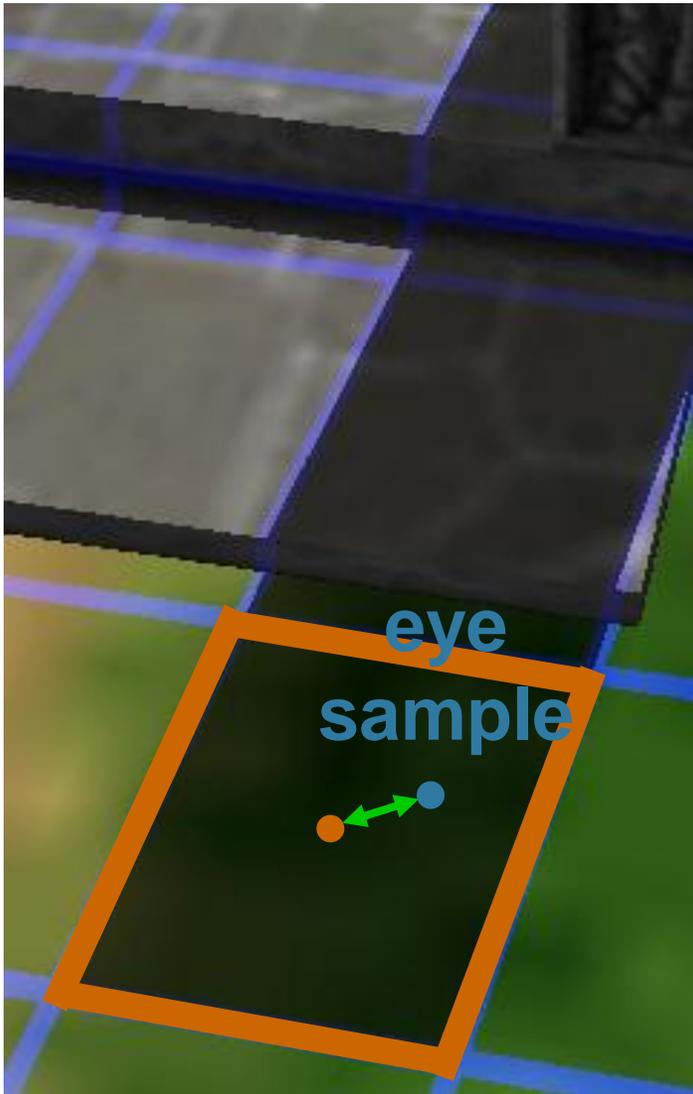


Temporal Smoothing

- Shadowing result of previous frame ($n-1$) is stored in *cache*
- Current frame (n): $cache(n) := w * s(n) + (1-w) * cache(n-1)$



Weight? Confidence Estimation!

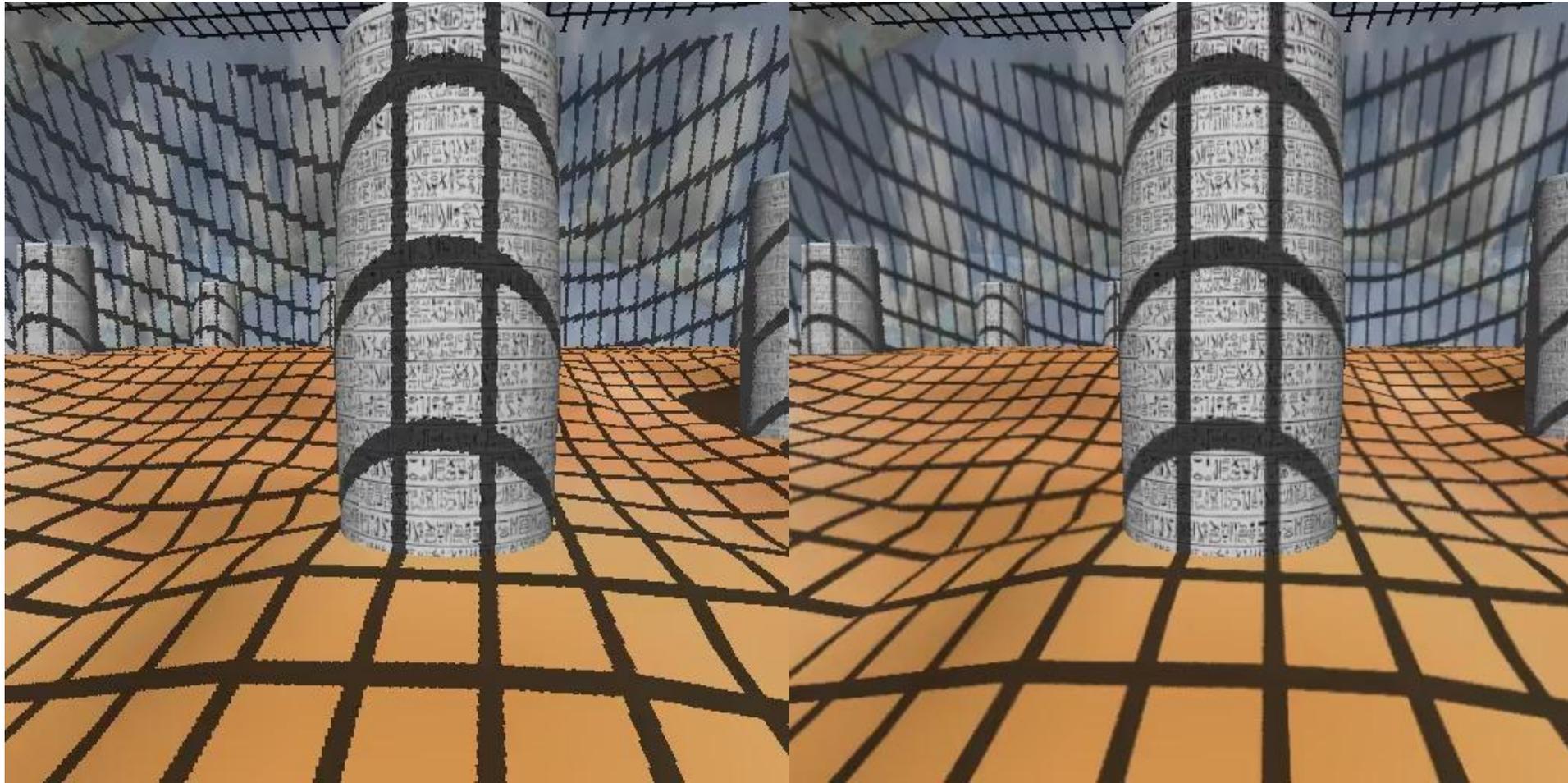


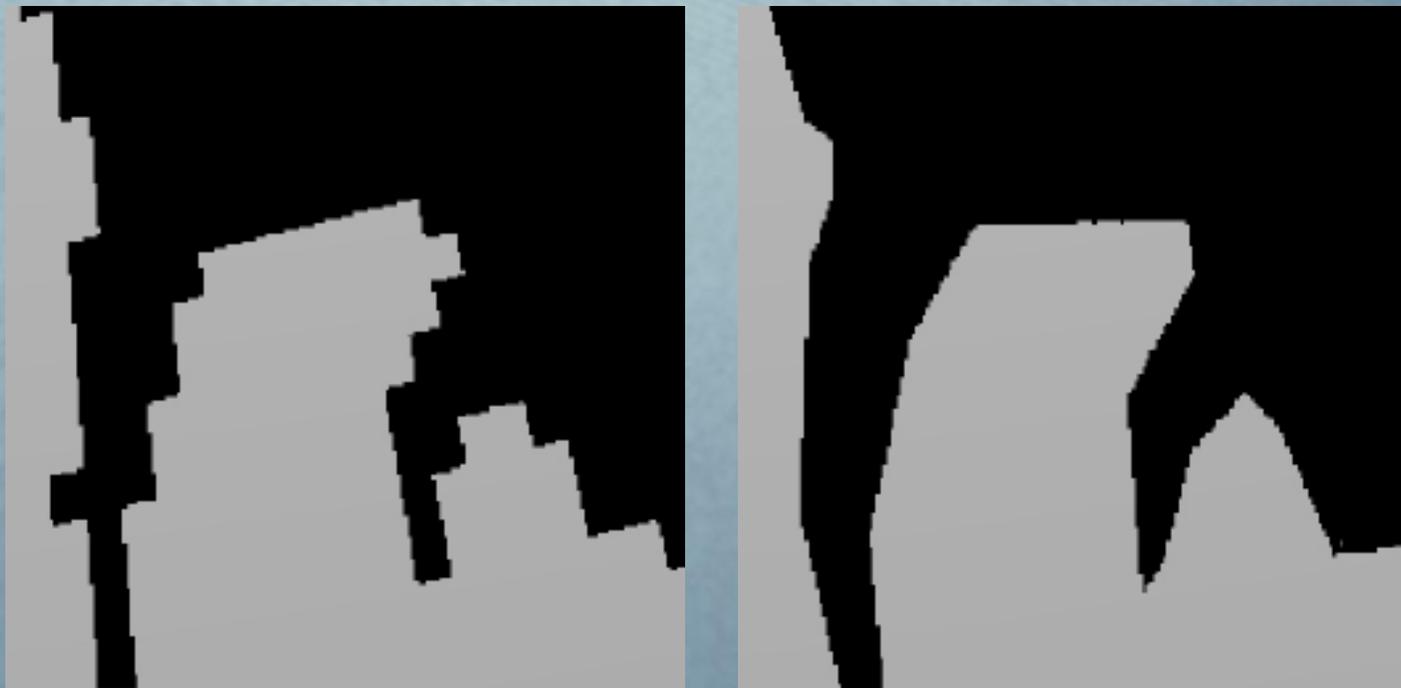
- Shadow map test probably more correct closer to texel center
- Confidence:
 $conf := 1 - dist(\text{eye}, \text{texel})$
- Greater confidence
→ greater impact

Confidence and Temporal Smoothing

- Confidence is weight for temporal smoothing

$$cache(n) := \mathbf{conf} * s(n) + (1 - \mathbf{conf}) * cache(n-1)$$





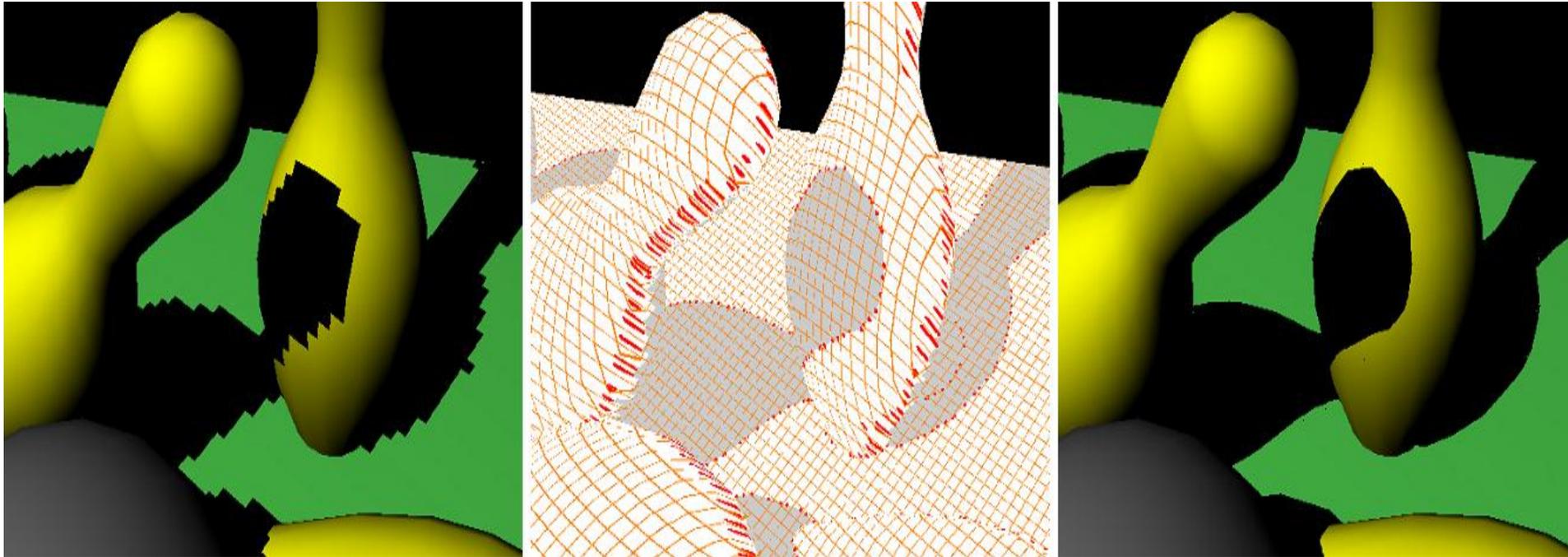
Hard Shadows

Better Reconstruction

Shadow Silhouette Maps

[Sen et al. 2003]

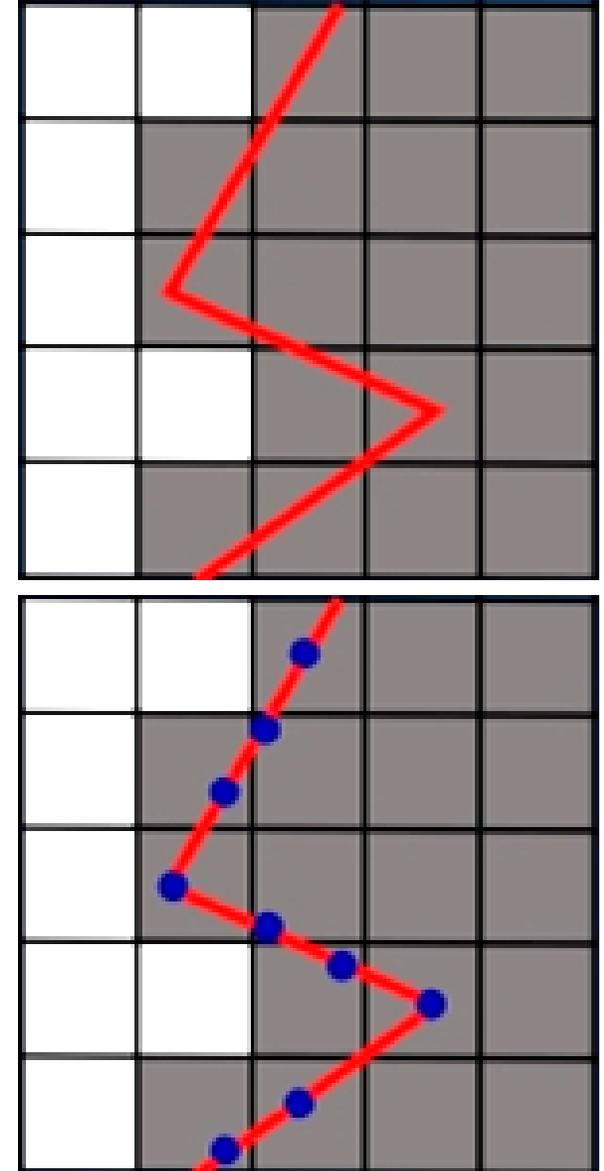
- Use a better silhouette approximation
- Store additional information of shadow edge



Shadow Silhouette Maps

[Sen et al. 2003]

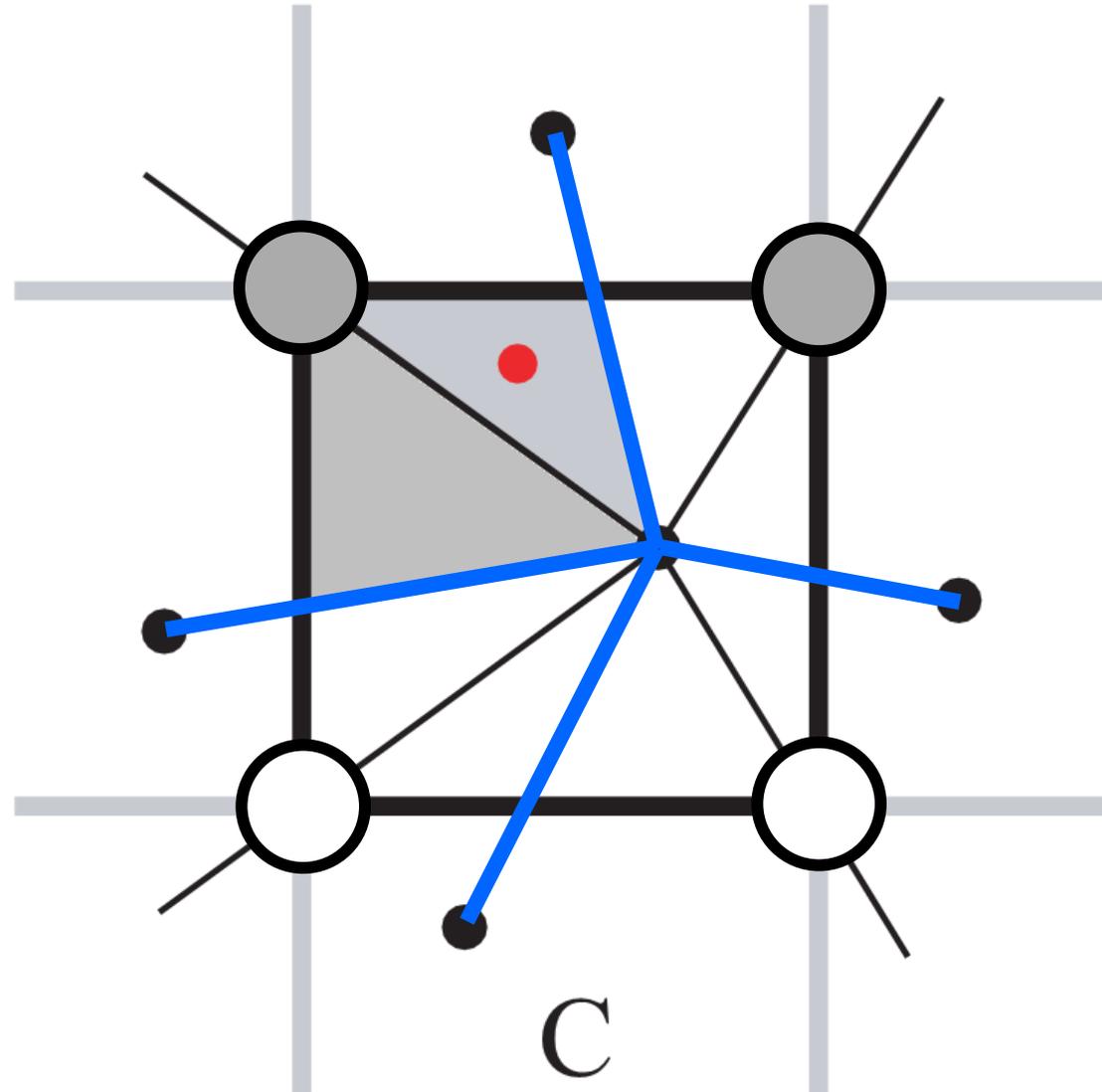
1. Create shadow map
2. Find silhouette edges
3. Rasterize silhouettes
 - a. Find points that lie on silhouette edges
 - b. Store such points into silhouette map
4. Compute shadows
 - a. Non-silhouette pixels use standard shadow map
 - b. Silhouette pixels use silhouette map



Shadow Silhouette Maps

[Sen et al. 2003]

- **Fragment**
- Silhouette points
 - 1 interior + 4 external
- Create quadrants
- Shade **fragment** according to shadow test result at corner point



Shadow Silhouette Maps

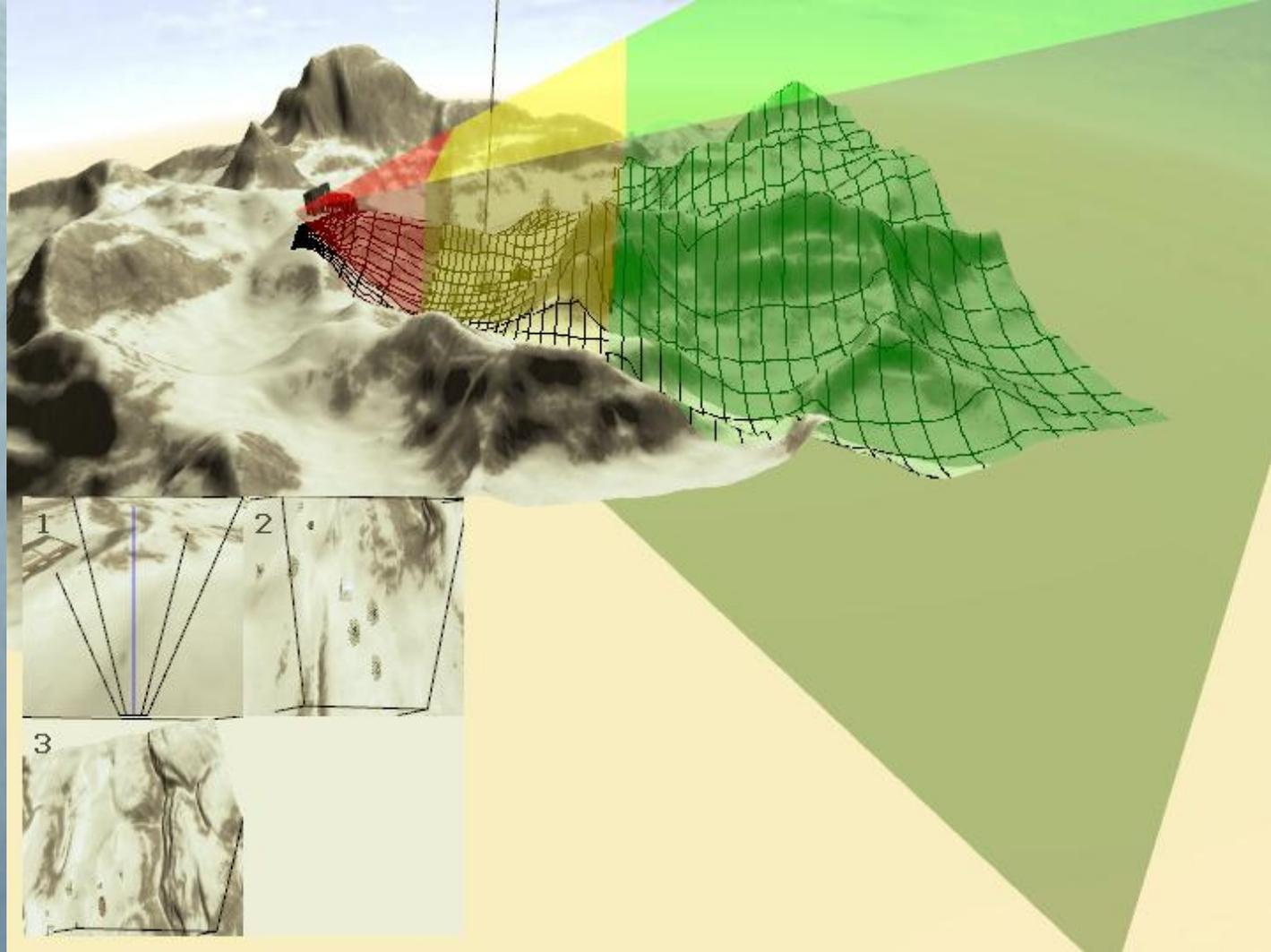
[Sen et al. 2003]



shadow map



silhouette map



Hard Shadows

Conclusion

- Fastest speed, single shadow map: **warping**
 - Good for outdoor
- Fast speed, better quality, >1 shadow maps:
z-partitioning (+warping)
 - Add “lightweight” scene analysis to tightly bound partitions and cull shadow casters
- High quality, lower speed: **adaptive partitioning**
- Reference quality, even slower: **irregular sampling**
 - Combine with subpixel sampling